KINGS

COLLEGE OF ENGINEERING
Punalkulam

DEPARTMENT OF MATHEMATICS
ACADEMIC YEAR – 2010 – 11(ODD SEMESTER)

QUESTION BANK

SUBJECT NAME: DISCRETE MATHEMATICS **YEAR/SEM: III/V**
SUBJECT CODE: MA1301

UNIT – I
PROPOSITIONS

PART – A

1. Define Converse, Inverse and Contrapositive of the statement?
2. Define atomic statement. What are the possible truth values for this statement?
3. Express the statement, “The crop will be destroyed if there is a flood” in symbolic form.
4. Write the negation of the following proposition. “To enter into the country you need a passport or a voter registration card”.
5. State the truth table of “If tigers have wings then the earth travels round the sun”.
6. Construct the truth table for (a) $\neg(\neg P \vee \neg Q)$ (b) $\neg(\neg P \wedge \neg Q)$
7. Define Tautology and Contradiction.
8. Give inverse and the contra positive of the implication “If it is a raining then I get wet”.
9. Write the following statement in symbolic form “If either Ram takes calculus or Krishna takes sociology, then Sita will take English.
10. Using truth table verify that the proposition $(P \wedge Q) \wedge \neg(P \vee Q)$ is a contradiction.
11. Prove that $(P \rightarrow Q)$and its contrapositive $(\neg Q \rightarrow \neg P)$are equivalent.
12. If P,Q and R are statement variable prove that $P \wedge ((\neg P \wedge Q) \vee (\neg P \wedge \neg Q)) \Rightarrow R$
13. Show that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$use only notation.
14. Write an equivalent formula for $p \wedge (q \leftrightarrow r)$which contains neither the biconditional nor the conditional.
15. Prove that whenever $A \wedge B \Rightarrow C$, we also have $A \Rightarrow (B \rightarrow C)$and vice versa.
16. Obtain disjunctive normal forms of $P \wedge (P \rightarrow Q)$
17. Define P.D.N.F. and P.C.N.F.
18. Define Minterms?
19. When a set of formulae is consistent and inconsistent?
20. Define valid arguments or valid conclusion.

PART - B

1. (a)Show that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$ using truth table. **(8)**
   (b)Without using truth tables, show that $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a tautology. **(8)**

2. (a) Prove that $(P \to Q) \wedge (Q \to R) \Rightarrow (P \to R)$. **(8)**
   (b)Obtain the DNF and CNF for $(P \to (Q \wedge R)) \wedge (\neg P \to (\neg Q \wedge \neg R))$ **(8)**
3. (a)Obtain PDNF and PCNF of $P \to ((P \to Q) \wedge \neg(\neg P \vee \neg Q))$ **(8)**
   (b)Obtain the PDNF for $(P \wedge Q) \vee (\neg P \wedge Q) \vee (Q \wedge R)$. **(8)**
4. Obtain the principal conjunctive normal form of the formula $(\neg P \to R) \wedge (Q \leftrightarrow P)$. **(16)**
   (a) Using truth table.
   (b) Without using truth table
5. (a) Find the principal conjunctive and principal disjunctive normal forms of the formula $S \Leftrightarrow (P \to (Q \wedge R)) \wedge (\neg P \to (\neg Q \wedge \neg R))$ **(8)**
   (b) Obtain the product of sums canonical form for $(P \wedge Q \wedge R) \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge \neg R)$ **(8)**
6. (a) Show that S is valid inference from the premises $P \to \neg Q, Q \vee R, \neg S \to P$ and $\neg$R **(8)**
   (b)Show that the following implication by using indirect method $R \to \neg Q, R \vee S, S \to \neg Q, P \to Q \Rightarrow \neg P$. **(8)**
7. (a) Determine whether the compound proposition $\neg(q \to r) \wedge r \wedge (p \to q)$ is tautology or contradiction **(8)**
   (b)Show that the premises $A \to (B \to C), D \to (B \wedge \neg C), A \wedge D$ are inconsistent. **(8)**
8. (a) Show that $J \wedge S$ logically follows from the premises $P \to Q, P \to \neg R, R, P \vee (J \wedge S)$. **(8)**
   (b) Using conditional proof prove that $\neg P \vee Q, \neg Q \vee R, R \to S \Rightarrow P \to S$. **(8)**
9. (a) Show that $R \vee S$ is a valid conclusion from the premises $C \vee D, (C \vee D) \to \neg H, \neg H \to (A \wedge \neg B)$ and $(A \wedge \neg B) \to (R \vee S)$ **(8)**
   (b) Show that d can be derived from the premises $(a \to b) \wedge (a \to c), \neg(b \wedge c), (d \vee a)$ **(8)**
10. (a) If there was a ball game, then traveling was difficult. If they arrived on time, then traveling was not difficult. They arrived on time. Therefore, there was no ball game". Test the validity of the above argument. **(8)**
    (b)Test the validity of the following argument, If I study, then i will not pass in the examination. If i watch TV, then i will not study. I failed in the examination. Therefore I watched TV. **(8)**

UNIT – II
PREDICATE CALCULUS

PART-A

1. What are free and bound variables in predicate logic.
2. Give Converse, Inverse and Contrapositive of a statement of the form, $(\forall x)(p(x) \to q(x))$
3. Define the term Universal Quantifier and Existential quantifier.
4. Symbolize the following statement with and without using the set of positive integers as the universe of discourse. "Give any positive integer, there is a greater positive Integers".
5. Let Q(x,y) denote the statement " x= y +2 ", what are the truth values of the propositions Q(1,2) and Q(2,0).
6. Find the truth value of (x)(P(x)vQ(x)) where P(x):x=1; Q(x):x=2 and the universe is {1,2}
7. Let the Universe of discourse be E={5,6,7}. Let A={5,6} and B={6,7}. Let P(x):x is in A; Q(x): x is in B and R(x,y):x+y<12. Find the truth value of $((\exists x)(P(x) \to (Q(x)) \to R(5,6)$.
8. If S = { -2,-1,0,1,2}, determine the truth value of $\forall x \in S, |x|^2 \leq 3|x| - 2$.

9. Give an example to show that $(\exists x)(A(x) \wedge B(x))$ need not be a conclusion form $(\exists x)A(x)$ and $(\exists x)B(x)$.

10. Define a compound statement function.
11. Express the statement " For every 'x' there exist a 'y' such that $x^2 + y^2 \geq 100$ ".
12. Express the statement, "Some people who trust others are rewarded" in symbolic form
13. Write the statement,"every one who likes fun will enjoy each of these plays" in symbolic form.
14. Express the statement " x is the father of the mother of y " in symbolic form.
15. Give the symbolic form of the statement "every book with a blue cover is a mathematics book".
16. Write in Symbolic form the statement."x is the brother of the sister of y"
17. Rewrite the following using quantifiers. " Some men are genius".
18. Write the following statement in the symbolic form " every one who likes fun will enjoy each of these plays".
19. Symbolize the expression " All the world lovers a lover"
20. Show that, $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x)P(x)$

PART-B

1. (a) Explain the two types of quantifiers and determine the truth table of each of the following statement: **(8)**
   (1) $\forall x, |x| = -x$
   (2) $\forall x, x + 2 > x$
   (3) $\exists x, x^4 = x$
   (4) $\exists x, x - 2 = x$

   (b) Find the scope of the quantifiers and the nature of occurrence of the variables in the formula $(\forall x)(P(x) \to (\exists y)R(x, y))$ **(8)**
2. (a)Show that $(x)A(x) \to (x)B(x) \Rightarrow (x)(A(x) \vee B(x))$ **(8)**

   (b) Show that following implication $(x)(P(x) \vee Q(x)), (x)(R(x) \to \neg Q(x)) \Rightarrow (x)(R(x) \to \neg P(x))$ **(8)**
3. (a) Prove that $(\exists x)P(x) \to (x)Q(x) \Rightarrow (x)(P(x) \to Q(x))$ **(10)**

   (b) Show that $(\exists x)M(x)$ follows logically from the premises $(x)(H(x) \to M(x))$ and $(\exists x)H(x)$ **(6)**
4. (a)Express the negations of the following statement using quantifiers and in statement form." No one has done every problem in the exercise". **(8)**

   (b) Show that from $(\exists x)(F(x) \wedge S(x)) \to (y)(M(y) \to W(y))$ and $(\exists y)(M(y) \wedge \neg W(y))$ the conclusion $(x)(F(x) \to \neg S(x))$ follows. **(8)**
5. (a) Show that $\neg P(a,b)$ follows logically from $(x)(y)(P(x, y) \to W(x, y))$ and $\neg W(a,b)$ **(8)**

   (b) Verify the validity of the inference. If one person is more successful than another, then he has worked harder to deserve success. John has not worked harder than Peter. Therefore, John is 'not successful than Peter. **(8)**
6. (a) Use indirect method of proof to show that **(8)**
   $(\forall x)(P(x) \vee Q(x)) \Rightarrow (\forall x)P(x) \vee (\exists x)Q(x)$

   (b) Prove that $(\exists x)P(x) \to (x)Q(x) \Rightarrow (\forall x)(P(x) \to Q(x))$ **(8)**

7. (a) Use indirect method of proof to show that $(\exists x)(A(x) \wedge B(x)) \Rightarrow (\exists x)A(x) \wedge (\exists x)B(x)$ **(8)**

   (b) Prove that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x)(P(x)) \wedge (\exists x)(Q(x))$ **(8)**

8. (a) Is the follwing argument valid?
   All lecturers are determined.
   Anyone who is determined and intelligent will give satisfactory service.
   Clare is an intelligent lecturer.
   Therefore Clare will give satisfactory service(use predicates) **(12)**

   (b) Use conditional proof to prove that $(x)(P(x) \rightarrow Q(x)) \Rightarrow (x)P(x) \rightarrow (x)Q(x)$. **(4)**

9. (a) Prove that $(\exists x)A(x) \rightarrow B \Leftrightarrow (x)(A(x) \rightarrow B)$. **(8)**

   (b) Verify the validity of the following argument:
   Lions are dangerous animals. There are lions. Therefore there are dangerous animals. **(8)**

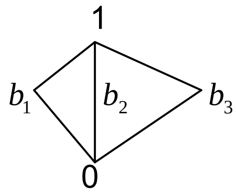
10. (a) Is the following conclusion validly derivable from the premises given? **(8)**
    If $(\forall x)(P(x) \rightarrow Q(x)), (\exists y)P(y)$ then $(\exists z)Q(z)$

    (b) Prove that (x)(H(x)$\rightarrow$A(x))$\Rightarrow$(x)($(\exists y)(H(y) \wedge N(x,y)) \rightarrow (\exists y)(A(y) \wedge N(x,y))$ **(8)**

UNIT –III
SET THEORY

PART-A

1. If A = { {1,2}, 3 }, B = { {1}, {2,3} } and C = { {1,2,3} } then Show that A,B and C are mutually disjoint.
2. Suppose that the sets A and B have m and n elements respectively. How many elements of A x B? How many different relations are there from A to B?
3. For any sets, A,B and C , prove that $AX(B \cap C) = (AXB) \cap (AXC)$.
4. Give an example of a relation. Which is not reflexive and not irreflexive?
5. Given an example of a relation which is symmetric, transitive but not reflexive on $\{a,b,c\}$.
6. If $A = \{2,3\} \subseteq X = \{2,3,6,12,24,36\}$ and the relation $\leq$ is such that $x \leq y$ is x divides y, find the least element and greatest element for A.
7. If $R = \{(1,1),(1,2),(1,3)\}$ and $S = \{(2,1),(2,2),(3,2)\}$ are relations on the set $A = \{1,2,3\}$, verify whether $R \circ S = S \circ R$ by finding the relation matrices of $R \circ S$ and $S \circ R$.
8. If A={1,2,3,4} and R = {(1,1),(1,3),(2,3),(3,2),(3,3),(4,3)}, determine the matrix of the relation R.
9. If a poset has a least element, then prove it is unique.
10. Define partially ordered set.
11. List all partitions of A = {1,2,3}.
12. Draw the Hasse-diagram of the set of partitions of 5.
13. Obtain the Hasse diagram of $(P(A_3), \subseteq)$, where $A_3 = \{a,b,c\}$.
14. Draw the Hasse diagram of $(X, \leq)$, where X is the set of positive divisors of 45 and the relation $\leq$ is such that $\leq$ $\{(x,y : x \in A, y \in A \wedge (x divides y)\}$
15. Draw the Hasse diagram of $D_{20} = \{1,2,4,5,10,20\}$
16. Partition $A = \{0,1,2,3,4,5\}$ with minsets generated by $B_1 = \{0,2,4\}$ and $B_2 = \{1,5\}$.
17. Give an example of a lattice which is modular but not a distributive.

18. Show that the lattice 

is not distributive.

19. Prove that $a.(a+b) = a + (a.b)$ in a Boolean Algebra.

20. Is the lattice of divisors of 32 a Boolean algebra?

PART-B

1. (a) Draw the graph of the relation R=$\{(x,y)/x,y \in X, x > y\} where X = \{1,2,3,4\}$ find the relation matrix **(8)**

   (b)Let R = $\{(1,1)(1,2),(1,3),(2,4),(3,2)\}$ and S=$\{(1,3),(1,4)(2,3),(3,1)(4,1)\}$ are the relation on A=$\{1,2,3,4\}$ obtain the relation matrices for R_0S,S_0R. **(8)**

2. (a) Given S=$\{1,2,3,4....10\}$ and a relation R on S where R=$\{(x,y)/x+y=10\}$ what are the properties of the relation R? **(8)**

   (b) Let R = $\{(1,2),(3,4),(2,2,)\}$ and S=$\{(4,2),(3,1),(1,3,)\}$ Find R_0S,S_0R,R_0 (S_0R),(R_0S) $_0R$,R_0R,S_0S and R_0R_0R **(8)**

3. (a) If R is the relation on the set of positive integers such that (a,b)$\in$R iff a^2+b^2 is even then prove that R is equivalence relation. **(8)**

   (b) Let the relation R be define on the set of all real numbers by 'if x,y are real numbers, $xRy \Leftrightarrow x$ - y is a rational numbers. Show that R is an equivalence relation. **(8)**

4. (a) Let P = {{1,2},{3,4},{5}} be a partition of the set S = { 1,2,3,4,5}. Construct an equivalence relation R on S so that equivalence classes with respect to R are precisely the members of P. **(4)**

   (b) If R is an equivalence relation on a set A, prove that $[x]=[y]$ if only if xRy where $[x]$ and $[y]$ denote equivalence classes containing x and y respectively. **(6)**

   (b) Define the relation P on $\{1,2,3,4,5\}$ by $P = \{(a,b)/|a-b| = 1\}$ determine the adjacency matrix of P^2 **(6)**

5. (a) Prove that R={(1,1)(1,4)(4,4)(2,2)(2,3)(3,2)(3,3)}is an equivalence relation. Also write the matrix of R and sketch its graph. **(8)**

   (b) Prove that, for any three sets A,B and C
      (i) A X (B U C) =(A X B) U (A X C)
      (ii) A U (B ∩ C) = (A U B) ∩ (A U C) **(8)**

6. (a) In a Lattice show that $a \le b$ and $c \le d$ implies $a * c \le b * d$. **(4)**

   (b) In a distributive lattice prove that a*b=a*c and $a \oplus b = a \oplus c$ implies that b=c **(8)**

   (c) Show that a chain with three or more elements is not complemented **(4)**

7. (a) If L is a distributive lattice with 0 and 1, show that each element has at most one complement. **(8)**

   (b) In a lattice (L,$\le$), prove that X$\vee$(Y$\wedge$Z)$\le (X \vee Y) \wedge (X \vee Z)$ **(8)**

8. (a) If B is a Boolean Algebra, then for $a \in B, a+1=1, a.0=0.$ **(8)**

(b) In any Boolean algebra show that $a = 0 \Leftrightarrow ab' + a'b = b.$ **(8)**

9. (a)Simplify the Boolean expression $((x_1+x_2)+(x_1+x_3)).x_1.\bar{x}_2$

(b) Establish De Morgan's Laws in a Boolean Algebra. **(8)**

10. (a) Show that in a Boolean algebra, for every element x has unique complement $\bar{x}$ such that $x+\bar{x}=1$ and $x.\bar{x}=0$ **(8)**

(b) In any Boolean algebra show that $a = b \Leftrightarrow ab' = a'b = 0.$ **(8)**

UNIT – IV
FUNCTIONS

PART-A

1. Find all the mappings from A = {1,2} to B = {3,4}
2. If A has m elements and B has n elements, how many functions are there from A to B.
3. If the function f is defined by $f(x) = x^2+1$ on a set A = {-2,-1,0,1,2}, find the range of f.
4. If $f: A \to B$ and $g: B \to C$ are mappings and $g \circ f : A \to C$ is one-to-one(Injection), prove that f is one-to-one.
5. Let $f: R \to R$ and $g: R \to R$ where R is the set of real numbers find $f \circ g$ and $g \circ f$, if $f(x) = x^2 - 2$ and $g(x) = x + 4.$
6. Let $h(x,y) = g(f_1(x,y), f_2(x,y))$ for all positive integers x and y, where $f_1(x,y) = x^2 + y^2, f_2(x,y) = x$ and $g(x,y) = xy^2$. Find h(x,y) in terms of x and y.
7. Show that the functions $f(x) = x^3$ and $g(x) = x^{1/3}$ for $x \in R,$ are inverse of one another.
8. Let f, g be functions from N to N where N is the set of natural numbers so that $f(n) = n+1, g(n) = 2n$. Determine $f \circ g$ and $g \circ f$.
9. The inverse of the inverse of a function is the function itself i.e., $(f^{-1})^{-1} = f.$
   (OR) If a function g be the inverse of a function f then f is the of g.
10. Show that $x * y = x - y$ is not a binary operation over the set of natural numbers, but that it is a binary operation on the set of integers. Is it commutative or associative?
11. Determine whether usual multiplication on the set $A = \{1,-1\}$ is a binary operation.
12. Examine whether matrix multiplication on the set $M = \left\{\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix} : a, b \in R\right\}$ is binary operation.
13. What are the identity and inverse elements under $*$ defined by $a * b = \frac{ab}{2} \forall a, b \in R.$
14. Define Characteristic function of a set.
15. Define the hashing function from the set of 8-digit account numbers 14739752 to the set {0,1,2,..........,100}.
16. If f(x,y) = x+y, express f(x,y+1) in terms of successor and projection functions.
17. Define primitive recursive function.
18. Show that $f(x,y) = x^y$ is primitive function.
19. Is the permutation p = $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 4 & 5 & 7 & 6 & 3 & 1 \end{pmatrix}$ even or odd?
20. Let A = {1,2,3,4,5,6,7,8}. Write the permutation (2,1,4,5,8,6) as the product of transpositions.

PART-B

1. a. Find all mappings from A = { 1 ,2, 3 } to B = { 4,5 }; find which of them are one-to-one and which are onto. **(8)**

   b. Let Z^+ denote the set of positive integers and Z denote the set of integers.

   Let $f : Z^+ \to Z$ be defined by $f(n) = \frac{n}{2}$, if n is even

   $= \frac{1-n}{2}$, if n is odd.

   Prove that f is a bijective and find f^{-1}. **(8)**

2. a. If : Z →N is defined by f(x) = $\begin{Bmatrix} 2x-1 : if x > 0 \\ -2x : if x \le 0 \end{Bmatrix}$. Prove that f is one-to-one and onto. **(8)**

   b. Let $a < b$. If $f : [a,b] \to [0,1]$ is defined by $f(x) = \frac{x-a}{a-b}$, prove that f is a bijection and find it inverse (Here $[a,b]$ and $[0,1]$ are closed intervals). **(8)**

3. a. Let A = {1,2,3}. If f is a function of A into itself defined by f(1)=2, f(2)=1, f(3)=3, find f^3 and f^{-1}. **(8)**

   b. Let $f : X \to Y$ and $f : Y \to X$. Prove that the function g is equal to f^{-1} only if $g \circ f = I_x$ and $f \circ g = I_y$ in the usual notation. **(8)**

4. a. If f and g are bijective on a set A , prove that $f \circ g$ is also a bijection.

   (or)The composite of two one-to-one and onto functions is also a one-to-one and onto function.

   (or)The composite of two bijections is also a bijection. **(8)**

   b. Let A,B,C be any three nonempty sets. Let $f : A \to B$ and $g : B \to C$ be mappings. If f and g are onto, prove that $g \circ f : A \to C$ is onto. Also give an example to show that $g \circ f$ may be onto but both f and g need not be onto. **(8)**

5.a. If f(x) = x+2, g(x) = x-2, h(x) = 3x for x∈R find $g \circ f, f \circ g, f \circ f, g \circ g, f \circ h, h \circ g, h \circ f, f \circ h \circ g$ where R is the set of real numbers. **(8)**

   b. If R denotes the set of real numbers and $f : R \to R$ is given by $f(x) = x^3 - 2$, find f^{-1}. **(4)**

   c. Let $f : R \to R$ be defined by $f(x) = 2x - 3$. Find a formula for f^{-1}. **(4)**

6. a. If A and B are any two subsets of a universal set U and ψ_A and ψ_B are the characteristic functions of A and B respectively. Show that $\psi_{A \cup B}(X) = \psi_A(X) + \psi_B(X) - \psi_{A \cap B}(X)$ for all X∈R. **(8)**

   b. Show that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ by using characteristic function. **(8)**

7. a. If A and B are any two subsets of universal set U, then prove that $f_{A \cap B}(x) = f_A(x) f_B(x)$ for all x∈R **(8)**

   b. Show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ by using characteristic function. **(8)**

8. a. Let a and b be positive integers and suppose f is defined recursively as follows:

   $f(a,b) = 5,$ if a<b

   $= f(a-b, b+2) + a$ if a≥b .Find f(2,7), f(10,3), f(15,2). **(8)**

b. The function A(x,y) is defined by A(0,y) = y+1, A(x+1,0) = A(x,1) and A(x+1,y+1) = A(x,A(x+1,y)). Compute the value of A(2,2). **(8)**

9. a. Show that the function $f(x, y) = x + y$ is primitive recursive. **(8)**

b. Let D(x) denote the number of divisors of x. Show that D(x) is a primitive recursive function. **(8)**

10.a. If $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 5 & 1 & 3 \end{pmatrix}$ and $h = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 2 & 4 & 3 & 1 \end{pmatrix}$ are permutations on the set A = { 1, 2,3, 4 ,5 }find a permutation g on A such that $f \circ g = h \circ f$. **(8)**

b. If $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$ are permutations, prove that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$. **(4)**

c. Let $p_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 3 & 2 & 1 & 4 & 5 & 6 \end{pmatrix}$ and $p_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 3 & 2 & 1 & 5 & 4 & 7 \end{pmatrix}$ compute (i) $p_1 \circ p_2$ (ii) p_1^{-1} **(4)**

UNIT-V
GROUPS

PART-A

1. Define semigroup and monoid. Give an example of a semigroup which is not a monoid.
2. Give an example of a monoid which is not a group.
3. Define semi group homomorphism.
4. Define sub semi-group with an example.
5. Prove that the only idempotent element of a group is its identity element.
6. Show that if every element in a group is its own inverse, then the group must be abelian.
7. When will a set of element form a monoid?
8. If S denotes the set of positive integers ≤ 100, for $x, y \in S$,define $x * y = \min\{x, y\}$. Verify whether (S,*) is a Monoid assuming that * is associative.
9. Let F = {2,4,8….} = {x: x= 2^n, n$\in$N }. Is F is closed under a) multiplication b) addition
10. What do you call a homomorphism of a semigroup into itself?
11. If H is a subgroup of the group G, among the right cosets of H in G, prove that there is only one subgroup viz., H.
12. Show that the inverse of an element in a group (G,*) is unique.
13. State Lagrange’s theorem for finite groups. Is the converse true?
14. Define ring and given an example of an ring with zero-divisors.
15. Define normal subgroup of a group.
16. Show that $(Z_{5,}, +_5)$ is a cyclic group.
17. Find the all cosets of the sub group H = {1, -1 } in G = { 1 -1 , i , -i } with the operation multiplication.
18. If the minimum distance between two code words is 5, howmany errors can be detected and howmany errors can be corrected?
19. Find the minimum distance between the code words x=(1,0,0,1), y=(0,1,0,0) and z=(1,0,0,0).
20. Define Group Code.

PART-B

1. a. If * is the binary operation on the set R of real numbers defined by a*b=a+b+2ab, Show that (R,*) is a commutative semigroup. **(8)**
   b. For any commutative monoid (M,*) the set of idempotent element of M forms a sub monoid. **(8)**
2. a. Show that the set N of natural numbers is a semigroup under the operation x*y = max{x,y}. Is it a monoid? **(8)**
   b. Prove that a subset S($\neq, \phi$) is a sub group if and only if for pair of element $a, b \in S, a * b^{-1} \in S.$ **(8)**
3. a. Show that monoid homomorphism preserves the property of invertibility. **(8)**
   b. If S = N X N, the set of ordered pairs of positive integers with the operation * defined by (a,b) * (c,d) = (ab+bc,bd) and if f:(S,*)→(Q,+) is defined by f(a,b) = $\frac{a}{b}$, show that f is a semi-group homomorphism. **(8)**
4. a. Show that the mapping g from the algebraic system (S,+) to the system (T,X) define by g(a) = 3^a , where S is the set all rational numbers under addition + and T is the set of non-zero real numbers under multiplication operation X, is a homomorphism but not an isomorphism. **(8)**
   b. Find all the non-trivial subgroups of $(Z_6, +_6)$. **(8)**
5. a. Show that H = { [0],[4],[8] } is a subgroup of $(Z_{12}, +_{12})$. Also find the left Cosets of H in $(Z_{12}, +_{12})$. **(8)**
   b. If S is the set of all ordered pairs (a,b) of real numbers with the binary operation $\oplus$ defined by (a,b) $\oplus$ (c,d) = (a+c,b+d) where a,b,c,d are real,prove that (S, $\oplus$) is a commutative group. **(8)**
6. a.The intersection of any two subgroups of a group G is again a subgroup of G.– Prove. **(8)**
   b. State and prove Lagrange's theorem for finite group. **(8)**
7. a. If H is a subgroup of G such that $x^2 \in H$ for every $x \in G$ prove that H is a normal subgroup of G. **(8)**
   b. Prove that the intersection of two normal subgroups of a group G is also a normal subgroup. **(8)**
8. a. Let $f:(G,*) \rightarrow (H, \Delta)$ be group homomorphism.Then show that Ker(f) is a normal subgroup. **(8)**
   b. Let $\langle A, * \rangle$ be a group. Let H = { a / $a \in G$ and a*b = b * a $\forall b \in G$}. Show that H is a normal subgroup. **(8)**
9. a. Let $H = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$ be a parity check matrix. Find
   (i). The Hamming code generated by H.
   (ii). Find the minimum distance of the code.
   (iii). If 100110 is the received word find the corresponding transmitted code word. **(8)**
   b. Show that the (2,5) encoding function e: $B^2 \rightarrow B^5$ defined by e(00) = 00000, e(01) = 01110, e(10) = 10101 and e(11) = 11011 is a group code. Find also the minimum distance of this group code. **(8)**

10. If $H = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

is the parity check matrix, find the Hamming code generated by H (in which the first three bits represent information portion and the next four bits are parity check bits).If y=(0,1,1,1,1,1,0) is the received word find the corresponding transmitted code word. **(16)**