KINGS

COLLEGE OF ENGINEERING

DEPARTMENT OF MATHEMATICS
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SUBJECT CODE/NAME: MA1251- NUMERICAL METHODS
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UNIT - I
SOLUTION OF EQUATIONS AND EIGENVALUE PROBLEMS

PART-A (2 MARKS)

1. State the iterative formula for regula falsi method to solve f(x) = 0.
2. Find an approximate value of the root of x^3-3x+1 = 0 lying between 1 and 2 by regula falsi method.
3. Write Newton's formula to find the cube root of N.
4. State the order of convergence of Newton's Raphson method
5. Find an iterative formula to find $\sqrt{N}$ by Newton's method.
6. State the criterion for the convergence in Newton Raphson method..
7. State the fixed point iteration theorem.(or) If g(x) is continous in [a,b], then under what condition in [a,b]?
8. What is the order of convergence for fixed point iteration?
9. Solve x+y = 2, 2x+3y = 5 by Gauss Elimination method.
10. When Gauss-Elimination method fails?
11. Solve the following system of equations by Gauss – Jordan method.
    5x + 4y = 15, 3x + 7 y = 12.
12. Distinguish Gauss Elimination method and Gauss Jordan method.
13. Distinguish between direct and iterative (indirect) methods of solving simultaneous equations.
14. Write a sufficient condition for Gauss_seidal method to converge?
15. Compare Gauss-Jacobi and Gauss seidal methods.
16. Find the inverse of the coefficient matrix by Gauss-Jordan method
    5x – 2y = 10 , 3x + 4y = 12
17. Find the inverse of A = $\begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix}$ by Gauss-Jordan method.
18. Determine the largest eigenvalue and the corresponding eigen vector of the matrix $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ correct to two decimal places using power method?

19. Find the dominant eigenvalue of A = $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ by power method?

20. How will you find the smallest eigen value of a square matrix A?

PART – B

1. a. Find the positive root of x^3-2x-5 = 0 by the method of false position,correct to four decimals. **(8)**
   b. Solve the equation 3x + sinx – e^x = 0 by Regula falsi method. **(8)**
2. a. Solve the equation xe^x = 2 by Regula Falsi method. **(8)**
   b. Find by Newton's method , the root of e^x = 4x near x = 2 , correct four decimal places. **(8)**
3. a. Find the iteration formula to find $\sqrt{N}$ where N is a positive integer by Newton's method and hence find $\sqrt{11}$ **(8)**
   b. Derive a Newton-Raphson iteration formula for finding the cube root of a positive number N. Hence find $\sqrt[3]{12}$ **(8)**
4. a. Obtain an iteration formula, using N - R values to find the reciprocal of a given number N and hence find $\frac{1}{19}$, correction of 4 decimal places **(8)**
   b. Find the double root of $x^3 - x^2 - x - 1 = 0$ choosing with the initial value of 0.8 **(8)**
5. a. Find a real root of the equation $x^3+x^2-100 = 0$. **(8)**
   b. Find a real root of the equation cos x = 3x-1 correct to 5 decimal places by fixed point iteration method. **(8)**
6. a. Solve the system of equations 10x-2y+3z=23; 2x+10y-5z=-33; 3x-4y+10z=41 using Gauss – elimination method **(8)**
   b. Solve the following system of equation using Gauss – elimination method 2x + y + 4z = 12 , 8x -3y +2z = 20 , 4x +11y – z =33. **(8)**
7. a. Using Gauss-Jordan method, solve the following system of equations 3x+4y-7z =23, 7x-y+2z = -14, x+10y-2z = 33. **(8)**
   b. Using Gauss-Jordan method, solve the following system of equations 2x –y +2z = 8, -x +2y + z = 4, 3x + y -4z = 0 **(8)**
8. a. Solve the following system of equations by Gauss-Jacobi Method 27x+6y-z=85; x+y+54z=110; 6x+15y+2z=72. **(8)**
   b. Using Gauss – Seidel method, solve the following system. Start with x = 1, y = -2, z = 3 , x+3y+52z = 173.61; x-27y+2z = 71.31; 41x-2y+3z = 65.46 **(8)**
9. a. Solve the following system by Gauss – Seidel method 10x-5y-2z = -3; 4x-10y+3z = -3; x+6y+10z = 3 . **(8)**
   b. Using Gauss-Jordon method , find the inverse of the matrix

$$\begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \\ 1 & 2 & 2 \end{pmatrix}$$ **(8)**

10. a. Find the numerically largest eigen values of A = $\begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$ by power

method corresponding eigen vector (correct to 3 decimal places). Start with initial eigen value $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$. **(8)**

b. Determine the largest eigen value and the corresponding eigen vector correct to 3 decimal places, using power method for the matrix A = $\begin{pmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{pmatrix}$ **(8)**

UNIT II
INTERPOLATION AND APPROXIMATION

PART- A (2 MARKS)

1. Write the Lagrange's formula to find y if three sets of values $(x_{0,} y_0)$, (x_1, y_1) and $(x_{2,} y_2)$ are given.
2. Obtain the Lagrange's interpolating polynomial for the observed data of points (1,1),(2,1) and (3,-2).
3. Find the polynomial which takes the following values

X:	0	1	2
Y:	1	2	1

4. Given f(0) = -1, f(1) = 1 and f(2) = 4. Find the Newton's interpolating formula.
5. State any two properties of divided differences.
6. State Newton's divided difference interpolation formula?
7. What are the n^{th} divided differences of a polynomial of the n^{th} degree.
8. Show that the divided differences are symmetrical in their arguments.
9. Find the divided difference for the following data

X:	2	5	10
Y:	5	29	109

10. Find the divided differences of f(x) = x^2+x+2 for the arguments 1,3,6,11.
11. What is a cubic spline.
12. What is meant by natural cubic spline.
13. State the conditions required for a natural cubic spline.
14. Given f(0)=-2,f(1)=2 and f(2)=8.Find the root of the Newton's forward interpolating polynomial equation f(x)=0.
15. State Newton – Gregory forward difference interpolation formula
16. Find the value of Y at x =21 using Newton's forward difference formula from the following table:

X:	20	23	26	29
Y:	0.3420	0.3907	0.4384	0.4848

17. State Newton's backward difference interpolation formula.
18. Find the polynomial for the following data by Newton's backward difference formula:

X:	0	1	2	3
F(x)	-3	2	9	18

19. Find f(2.5) from the data:

X:	1	2	3
F(x)	0	1	8

20. Find the sixth term in the sequence 8,12,19,29,42,......

PART – B

1. a. Find the Lagrange's polynomial of degree 3 to fit the data: **(8)**
Y(0)= - 12, y(1)=0,y(3)=6 and y(4) =12. Hence find y(2).

b. . Using Lagrange's formula, fit a polynomial to the data **(8)**

X:	-1	0	2	3
Y:	-8	3	1	12

Hence find y at x=1.5 and x=1

2. a. Using Lagrange's formula, fit a polynomial to the data **(8)**

X:	0	1	2	4	5	6
f(x):	1	14	15	5	6	19

Also find f(3).

b**.** Using Lagrange's formula, find y at x=6 for the following data: **(8)**

X:	2	5	7	10	12
Y:	18	180	448	1210	2028

3. a. Find the age corresponding to the annuity value 13.6 given the table **(8)**

Age (x)	30	35	40	45	50
Annuity value (y)	15.9	14.9	14.1	13.3	12.5

b. If f(0) = 0 , f(1) = 0, f(2) = -12, f(4) =(0), f(5) =600 and f(7) = 7308,find a polynomial that satisfies this data using Newton's divided difference interpolation formula. Hence, find f(6). **(8)**

4. a. Using Newton's divided difference formula find f(x) and f(6) from the following data: **(8)**

X:	1	2	7	8
f(x)	1	5	5	4

b. Using Newton's divided difference formula to find f(5) from the following data **(8)**

X:	0	2	3	4	7
f(x)	4	26	58	112	466

5. a. Using Newton's divided difference formula find the cubic function of x. from the following data: **(8)**

X:	0	1	4	5
f(x)	8	11	68	123

b. Using Newton's divided difference formula find the cubic function of x from the following data: **(8)**

X:	0	1	4	5
f(x)	2	3	12	147

6. Using cubic spline, find y(0.5) and y(1.5) from the following data, assuming that y''(0)=0 and y''(2)=0 **(16)**

x	0	1	2
y	-5	-4	3

7. Find the cubic spline for the data:

x	1	2	3	4
f(x)	1	2	5	11

Assume that y''(1)=0 and y''(4)=0. **(16)**

8. a. Find the polynomial of degree two for the data by Newton's forward difference method: **(8)**

X:	0	1	2	3	4	5	6	7
F(x)	1	2	4	7	11	16	22	29

b. From the following table of half-yearly premium for policies maturing at different ages, estimate the premium for policies maturing at age 46 and 63. **(8)**

Age X:	45	50	55	60	65
PremiumY:	114.84	96.16	83.32	74.48	68.48

9. a. From the following table,find the value of tan(0.12) **(8)**

x:	0.10	0.15	0.20	0.25	0.30
y=tan x	0.1003	0.1511	0.2027	0.2553	0.3093

b. Use Newton's backward difference formula to construct an interpolating polynomial of degree 3 for the data **(8)**

f(-0.75) = -0.07181250 , f(-0.5) = -0.024750

f(-0.25) = 0.33493750 , f(0) = 1.10100 Hence find $f(-\frac{1}{3})$.

10. a. Given **(8)**

X:	1	2	3	4	5	6	7	8
f(x):	1	8	27	64	125	216	343s	512

Estimate f(7.5). Use Newton's formula.

b. The following data are taken from the steam table: **(8)**

Temp.C	140	150	160	170	180
Pressure kgf/cm^2	3.685	4.854	6.302	8.076	10.225

Find the pressure at temperature at t=175.

UNIT – III
NUMERICAL DIFFERENTIATION AND INTEGRATION

PART – A (2 MARKS)

1.State Newton's formula to find f'(x) using the forward differences.

2. Find $\frac{dy}{dx}$ at x = 1 from the table

x : 1 2 3 4

y : 1 8 27 64

3. Using Newton's backward difference formula, write the formulae for the first and second order derivatives at the end values x = x_n upto the fourth order difference term.
4. Find $y'(5)$ from the following table

x :	0	1	2	3	4	5
y :	4	8	15	7	6	2

5. State the formula for Trapezoidal rule of integration

$$\int_{x_0}^{x_n} f(x)dx. = \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + ... + y_{n-1})]$$

6. A curve passes through (2,8), (3,27), (4,64) and (5,125) Find the area of the curve between the x axis and the lines x = 2 and x = 5 by Trapezoidal rule.
7. Given f(0)=-1,f(1)=1 and f(2)=4, find $\int_0^2 f(x)dx$ by Trapezoidal rule.
8. Evaluate $\int_0^1 \frac{dx}{1+x}$ with h = 0.5 using Trapezoidal rule.
9. What is the order of trapezoidal rule ?
10. Using Simpson's rule find $\int_0^4 e^x dx.$ given $e^0 = 1$, $e^1 = 2.72$, $e^2 = 7.39$, $e^3 = 20.09$ and $e^4 = 54.6$.
11. What is the local error term in Trapezoidal formula and in Simpson's one third rule?
12. Find $\int_{-2}^{2} x^4 dx.$ by Simpson's rule taking h = 1. ..
13. Compare Trapezoidal rule and Simpson's 1/3 rule for evaluating numerical integration.
14. State Romberg's method integration formula to find the value of I = $\int_a^b f(x)dx.$ using h and h/2.
15. If $I_1 = 0.775$, $I_2 = 0.7828$ find I using Romberg's method.
16. If I = $\int_0^1 e^{-x2}dx.$ then $I_1 = 0.731$, $I_2 = 0.7430$ with h = 0.5 and h = 0.25 Find I using Romberg's method.
17. Compute $\int_{-2}^{2} e^{\frac{-x}{2}} dx.$ using Gaussian two point formula.
18. State three point Gaussian quadrature formula.
19. State Trapezoidal rule for evaluating $\int_a^b \int_c^d f(x,y)dxdy.$

20. State Simpson's rule for evaluating $\int_a^b \int_c^d f(x,y)dxdy.$

PART– B

1. a. Find the value of f '(8) from the following table **(8)**

x :	6	7	9	12
$f(x)$:	1.556	1.690	1.908	2.158

b. Find the first and second derivative of the function tabulated below at x = 0.4

x :	0.4	0.5	0.6	0.7	0.8
y :	1.5836	1.7974	2.0442	2.3275	2.6511

(8)

2. a. Find $\frac{dy}{dx}$ at x = 1.5 given

x :	1	2	3	4	5
y :	77	78	127	248	375

(8)

b. Find the first and second derivatives of y w. r. to x at x = 10

x :	3	5	7	9	11
y :	31	43	57	41	27

(8)

3. a. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x=10 for the following data **(8)**

x:	2	4	6	8	10
y:	6	54	134	246	390

b. Find the maximum and minimum value of y = f(x) given the data

x :	0	1	2	3	4	5
f(x) :	0	$\frac{1}{4}$	0	$\frac{9}{4}$	16	$\frac{225}{4}$

(8)

4. a. A river is 80 mts wide. The depth 'd' in mts at a distance x mts from one bank is given by the following table. Calculate the area of cross section of the river using Simpson's $\frac{1}{3}$rd rule

x :	0	10	20	30	40	50	60	70	80
d :	0	4	7	9	12	15	14	8	3

(8)

b. By dividing the range into equal parts, evaluate $\int_0^\pi \sin x dx$ by using Simpson's $\frac{1}{3}$rd rule. **(8)**

5. a. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by Trapezoidal rule and simpson's 3/8 rule. **(8)**

b. Calculate $\int_{0.5}^{0.7} e^{-x}\sqrt{x}dx$ taking 5 ordinates by trapezoidal rule and Simpson's1/3 rule **(8)**

6. a. Evaluate $\int_1^{1.4} e^{-x^2} dx$ by taking h=0.1using Simpson's 3/8 rule. **(8)**

b. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Romberg's method. Hence obtain an approximate value of Π. **(8)**

7. a. Evaluate $\int_0^2 \frac{dx}{x^2+4}$ using Romberg's method. Hence obtain an approximate value of Π. **(8)**

b. Evaluate $\int_3^7 \frac{dx}{1+x^2}$ using Gaussian Quadrature with three points. **(8)**

8. a. Evaluate I = $\int_2^3 \frac{dt}{1+t}$ by Three point Gaussian Quadrature formula. **(8)**

b. Evaluate $I = \int_0^1 \frac{dx}{1+x}$ using two point Gaussian Quadrature formula. **(8)**

9. a. Evaluate $\int_1^2\int_1^2 \frac{dxdy}{x+y}$ by using Simpson's $\frac{1}{3}$ rd rule taking Δx = Δy = 0.25 **(8)**

b. Evaluate $\int_1^2\int_3^4 \frac{1}{(x+y)^2}dxdy$ taking h = k = 0.5 by both trapezoidal and Simpson's rule **(8)**

10. a. Evaluate $\int_1^{1.2}\int_1^{1.4} \frac{1}{(x+y)}dxdy$, by using Trapezoidal rule h = 0.1 and k = 0.1 **(8)**

b. Evaluate $\int_0^{\pi/2}\int_{\pi/2}^{\pi} \cos(x+y)dxdy$ by using Trapezoidal rule, Simpson's 1/3 rule and taking h=k=$\frac{\pi}{4}$. **(8)**

UNIT – IV
INITIAL VALUE PROBLEMS FOR ORDINARY DIFFERENTIAL EQUATIONS

PART – A (2 MARKS)

1. State the disadvantage of Taylor series methods?
2. Write the merits and demerits of the Taylor method of solution?
3. State Taylor series algorithm for the first order differential equation?
4. Solve the differential equation $\frac{dy}{dx} = x + y + xy, y(0) = 1$ by Taylor series method to get the value of y at x = h?
5. Write down Euler method to the differential equation $\frac{dy}{dx} = f(x, y)$.
6. State modified Euler method to solve y ' = f(x,y), $y(x_0)=y_0$ at x = x_0 + h.
7. Using Modified Euler's method, find y(0,1) if $\frac{dy}{dx} = x^2 + y^2, y(0) = 1$
8. What is the error of Euler's method?

9. What are the limitations of Euler's method?
10. What is the Error in modified Euler's method?
11. Write down the Runge-Kutta formula of fourth order to solve dy/dx = f(x,y) with $y(x_0) = y_0$.
12. State the special advantage of Runge-Kutta method over Taylor series method? (or)
    Compare Taylor's series and R.K. method?
13. Write Milne's predictor corrector formula? .
14. How many prior values are required to predict the next value in Milne's method & Adam's method?
15. Compare Runge-kutta methods and Predictor-Corrector methods for solution of Initial value problem.
16. What is the error term in Milne's corrector formula?
17. What is the error term in Milne's predictor formula?
18. Write down Adams-Bashforth formula?
19. What is a Predictor-Collector method of solving a differential equation?
20. What is the Error of Adam Bashforth method?

PART – B

1. a. Using Taylor series method find y at x = 0.1 if $\frac{dy}{dx} = x^2 y - 1, y(0) = 1.$ **(8)**

   b. Solve $\frac{dy}{dx} = x^2 + y^2, y(0) = 1.$ Use Taylor series at x=0.2 and 0.4, Find x = 0.1. **(8)**

2. a. Solve the system of equations $\frac{dy}{dx} = z - x^2,\ \frac{dz}{dx} = y + x,$ with y(0) = 1,z(0) = 1 by taking h = 0.1 , to get y(0.1) and z(0.1).Here y and z are dependent variables and x is independent. **(8)**

   b. Using Taylor series method, find y(1.1) and y(1.2) correct to four decimal places. Given $\frac{dy}{dx} = xy^{1/3}$ and y(1) =1. **(8)**

3. a. By Taylor's series method find y(0.1) given that $y'' = y + xy'$, y(0) =1 , $y'(0) = 0$ **(8)**

   b.Using Eulers modified method find y(0.1) from $y' = x + y + xy, y(0) = 1,$ with h=0.05 **(8)**

4. a. Using Euler's method find y(0.3)of y(x) satisfies the initial value problem.

   $y' = \frac{1}{2}(x^2 + 1)y^2, y(0.2) = 1.1114,$ with h=0.1. **(8)**

   b. Using modified Eulers method, compute y(0.1) with h=0.1 from

   $\frac{dy}{dx} = y - \frac{2x}{y}, y(0) = 1.$ **(8)**

5. a. Solve $y' = 1 - y, y(0) = 0,$ find y(0.1) by modified Euler's method. **(8)**

   b. Given $\frac{dy}{dx} = x^3 + y, y(0) = 2.$ Compute y(0.2), y(0.4) and y(0.6) by Runge-Kutta method of fourth order. **(8)**

6. a. Using R.K. method of 4^{th} order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}, y(0) = 1.$ at x = 0.2. **(8)**

b. Using R.K. method of fourth order find y(0.1) for the initial value problem $\frac{dy}{dx}=\frac{xy}{1+x^2}, y(0)=1.$,take h=0.1. **(8)**

7. Determine the value of y(0.4) using Milne's method given $\frac{dy}{dx}=y^2+xy, y(0)=1;$. use Taylor series to get the values of y(0.1), y(0.2) and y(0.3). **(16)**
8. Given $y' = 1-y$ and y(0) = 0 , find
   (i) y(0.1) by Euler method
   (ii) y(0.2) and y(0.3) by modified Euler method
   (iii) y(0.4) by Milne's method **(16)**
9. a. Using Milne's method find y(4.4) given $5xy'+y-2=0$ given y(4)=1, y(4.1) =1.0049, y(4.2)=1.0097 and y(4.3) = 1.0143. **(8)**

   b. Given $\frac{dy}{dx}=x^2(1+y), y(1)=1, y(1.1)=1.233, y(1.2)=1.548, y(1.3)=1.979$, evaluate y(1.4) by Adams – Bashforth method. **(8)**
10. Consider the initial value problem $\frac{dy}{dx}=y-x^2+1, y(0)=0.5.$ **(16)**
    a. using Taylor series, find y(0.2)
    b. Using 4^{th} order Runge-Kutta method, find y(0.4) and y(0.6)
    c. Using Adam-Bashforth Predictor.

UNIT V
BOUNDARY VALUE PROBLEMS IN ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS

PART – A (2 MARKS)

1. State the conditions for the equation. $Au_{xx} + Bu_{yy} + Cu_{xy} + Du_x + Eu_y + Fu = G$ where A, B, C, D, E, F, G are function of x and y to be (i) elliptic (ii) parabolic (iii) hyperbolic
2. State the condition for the equation $Au_{xx}+2Bu_{xy}+Cu_{yy}=f(u_x,u_y,x,y)$ to be (a)elliptic (b) parabolic(c) hyperbolic when A, B, C are functions of x and y.
3. What is the classification of $f_x-f_{yy}=0$? .
4. State Schmidt's explicit formula for sloving heat flow equation?
5. What is the classification of one dimensional heat flow equation?
6. Write down the Crank-Nicholson formula to solve $u_t=u_{xx}$?

   (or)

   Write down the implicit formula to solve one dimensional heat flow equation?
7. What type of equations can be solved by using Crank-Nicholson's difference formula?
8. Write the Crank Nicholson difference scheme to solve $u_{xx}=au_t$ with $u(0,t) = T_0$, $u(l,t)=T_1$ and the initial condition as u(x,0) = f(x)?

9. For what purpose Bender-Schmidt recurrence relation is used
10. State the explicit scheme formula for the solution of the wave equation?

11. For what value of λ, the explicit, method of solving the hyperbolic equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ is stable, where $\lambda = C\Delta t/\Delta x$?
12. Write the diagonal five-point formula to solve the Laplace equation $u_{xx}+u_{yy}=0$?
13. Write down the standard five point formula to solve Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.
14. What is the purpose of Liebmann's process?
15. If u satisfies Laplace equation and u = 100 on the boundary of a square what will be the value of u at an interior grid point?
16. State Liebmann's iteration process formula?
17. Write down the finite difference form of the equation $\nabla^2 u = f(x,y)$
18. State the general form of Poisson's equation in partial derivatives? $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x,y)$
19. What is Shooting method?
20. What is the procedure of shooting method?

PART - B

1. a. Solve by finite difference method, the boundary value problem $y''(x) - y(x) = 2$ where $y(0) = 0$ and $y(1) = 1$, taking $h = 1/4$. **(8)**

   b. Using the finite difference method, find y(0.25),y(0.5) and y(0.75) satisfying the differential equation $y''(x) + y = x$ subject to the boundary conditions y(0) = 0,y(1)= 2. **(8)**

2. a. Solve $\frac{\partial^2 u}{\partial x^2} = 2\frac{\partial u}{\partial t}$ given u(0,t)=0, u(4,t)=0,u(x,0)=x(4-x) taking $\Delta x = \Delta t = 1$. Find the value of u upto t=3 using Bender-Schmidt explicit difference scheme. **(8)**

   b. Using Schmidt's process solve $25\, u_{xx}=u_t$ where $0<x<1$ $t>0$ with boundary conditions u(0,t)=0=u(10,t); u(x,0)= $\frac{x(10-x)}{25}$ and choosing h=1 and k suitably. Find $u_{i,j}$ for i=1,2,3….9 and j= 1,2,3,4 **(8)**

3. a. Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 \le x \le 1$, $t \ge 0$ with u(x,0)=x(1-x), $0<x<1$ and u(0,t)=u(1,t) = 0, for all $t>0$ using explicit method with $\Delta x=0.2$ for 3 time steps. **(8)**

   b. Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ in $0<x<5$, $t\ge 0$ given that u(x,0) =20, u(0,t)=0,u(5,t) =100.Compute u for the time-step with h=1 by Cranck – Nicholson method **(8)**

4. a. Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$, $0<x<2, t>0$, u(0,t)=u(2,t)=0,t>0 and u(x,0)= $\sin\frac{\pi x}{2}$, $0 \le x \le 2$, and $\Delta t=0.25$ and $\Delta x=0.5$ for two times steps by Crank-Nicholson implicit finite difference method. **(8)**

   b. Solve $\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}$, $0<x<1$.$t>0$ given u(x,0)=0, $\frac{\partial}{\partial t}(x,0)$ =u(0,t)=0 and u(1,t)=100sinπt, complete u(x,t) for 4 times steps with h=0.25. **(8)**

5. Solve the Laplace's equation over the square mesh of side 4 units satisfying the boundary conditions**:** **(16)**

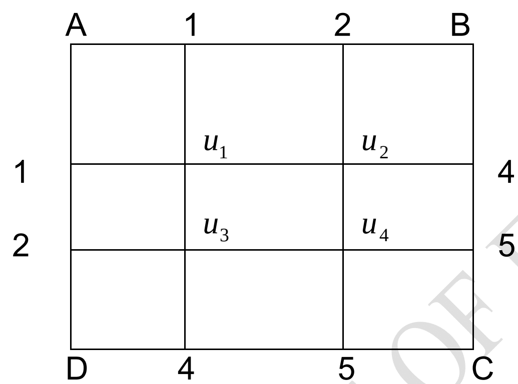
   $U(0,y)=0, 0 \le y \le 4$; $u(4,y)=12+y$, $0 \le y \le 4$

   $U(x,0)=3x$, $0 \le x \le 4$; $u(x,4)= x^2$, $0 \le x \le 4$

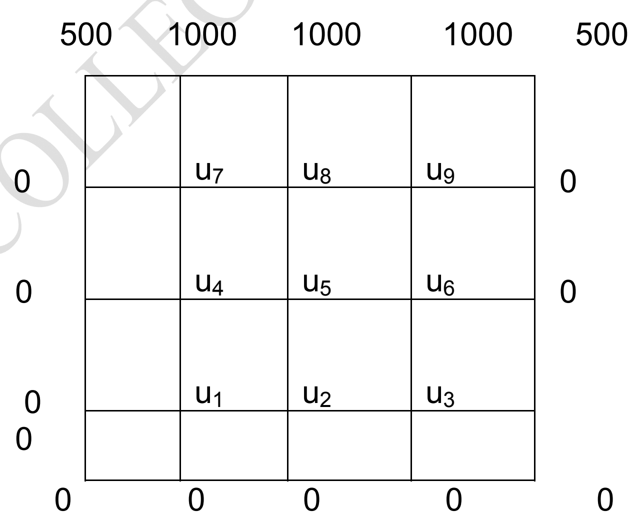
6. By iteration method , solve the laplace equation $u_{xx}+u_{yy}=0$, over the square region, satisfying the boundary condition. **(16)**

   $u(0, y) = 0, \quad 0 \le y \le 3$
   $u(3, y) = 9+y, \ 0 \le y \le 3$
   $u(x,0) = 3x, \quad 0 \le x \le 3$
   $u(x, 3) = 4x, \quad 0 \le x \le 3$

7. Solve $u_{xx}+u_{yy} = 0$ for the following square mesh with boundary values as shown in the figure below **(16)**



8. Solve the Laplace equation at the interior points of the square region given below **(16)**



9. Solve $\nabla^2 u = -10(x^2 + y^2 + 10.)$ over the square mesh with sides x=0,y=0,x=3,y=3 with u=0 on the boundary and mesh length of 1 unit**.** **(16)**

10. Solve the Poisson equation $u_{xx} + u_{yy} = -x^2 y^2$ over the square region bounded by the lines x=0,y=3 given that u=10 throughout the boundaries taking h=1**.** **(16)**