KINGS

COLLEGE OF ENGINEERING
DEPARTMENT OF MATHEMATICS
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QUESTION BANK

SUBJECT CODE/NAME: MA1201-TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS
YEAR/SEM: II / III

UNIT I
FOURIER SERIES

PART – A

1. Explain periodic function with two examples.
2. State Dirichlets condition
3. To which value,the half range sine series corresponding to $f(x) = x^2$ expressed in the interval (0,5)converges at x=5?
4. If f(x) = e^{ax} is expanded as a Fourier series in (0,2π) ,what is the value of b_n ?
5. Does f(x) = tan x posses a Fourier expansion?
6. Obtain the value of a_0 in the Fourier expansion of f(x) = $\sqrt{1-\cos x}$ in (0,2π)
7. In the Fourier expansion of f(x)= $1+\frac{2x}{\pi}$, $-\pi < x < 0$

   $= 1-\frac{2x}{\pi}$, $0 < x < \pi$ in (-π, π).

   Find the value of b_n, the coefficient of sin nx.
8. Determine the Value of a_0 of the function f(x)=$|\sin x|$,$-\pi < x < \pi$ in the Fourier expansion.
9. If f(x) = x + x^2 is expanded as a Fourier series in (-π,π), find the value of a_n .
10. Find the coefficient b_5 of cos5x, in the Fourier cosine series of the function f(x)=sin5x in the interval (0,2π)
11. f(x) = x(2π-x) is expanded as a Fourier series in (0,2π). Find a_n
12. State the nature of the Fourier expansion of f(x) = xcosh2x in (-π,π).
13. Without evaluating any integral , write the half range series with sine terms for $f(x) = \sin^3 x$ in (0,π).
14. Find the half range sine series of f(x)=xcosx in (0, π) .Find the value of b_1
15. Write the Complex form of the Fourier series of f(x).
16. Define root mean square value of a function f(x) in a<x<b.

17. Find the root mean square value of the function f(x) = x in the interval (0,l)
18. State Parseval's Theorem on Fourier series.
19. State Parseval's identity for the half range cosine expansion of f(x) in (0,1)
20. What do you mean by Harmonic Analysis?

PART – B

1.(a) Find the Fourier series for $f(x) = 1$ in (0, π)

= 2 in (π ,2 π)

and hence find the sum of the $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}$----∞ . **(8)**

(b)Obtain the Fourier series for $f(x) = 1 + x + x^2$ in the interval -π < x < π.

Deduce that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}---=\frac{\pi^2}{6}$ **(8)**

2.(a)Determine the Fourier series for the function = 1+x , 0 < x < π

= -1+x , - π < x < 0.

Hence deduce that $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+---=\frac{\pi}{4}$ **(8)**

(b) Find the fourier series of f(x)=x^2 in [0,2π] and periodic with period 2π.

Hence deduce that $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$. **(8)**

3.(a)Find the Fourier series for the function f(x) = x in 0<x<1

= 1 –x in 1 < x <2

deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}$----∞ = $\frac{\pi^2}{8}$. **(8)**

(b)Obtain Fourier series of period $2l$ for f(x)

where f(x) = l –x in $0 \le x \le l$

= 0 in $l \le x \le 2l$.

Hence find the sum of $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+---\infty$ **(8)**

4. (a)Obtain the Fouier series for the function f(x) =x,0<x<l,

=2l-x,l<x<2l . **(8)**

(b)Obtain the Fourier expansion of x sin x as a cosine series in (0, π)

and hence deduce the value of $1+\frac{2}{1.3}-\frac{2}{3.5}+\frac{2}{5.7}$..... **(8)**

5 . (a) Explain f(x)= (1+ cos x) ^{2} as Foruier cosine series in (0, π) **(8)**

(b)Find the half range sine and cosine series for the function f(x) = e^x. **(8)**

6. (a) Find the half range sine and cosine series for function f(x) = xcos x in (0, π). **(8)**

(b) By finding the fourier cosine series for $f(x) = x$ in 0<x< π,

show that $\frac{\pi 4}{96}=\sum_{n=1}^{\infty}\frac{1}{(2n-1)4}$ **(8)**

7 (a) Find the half range sine series for f(x) = $(\pi - x)^2$ in the interval (0, π).

Hence find the Sum of the series $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}---\infty$ **(8)**

(b) Find the half range cosine series for f(x) =x (π –x) in the interval (0, π) and deduce that $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}---\infty=\frac{\Pi^4}{90}$ **(8)**

8.(a)Find the complex form of the Fourier series f(x)=cos ax in $-\pi<x<\pi$. **(8)**

(b)Find the complex form of the Fourier series f(x)=e^{ax} in (-l,l). **(8)**

9.(a)Find the Fourier series as the second Harmonic to represent the function given in the following data

x	0	1	2	3	4	5
y	9	18	24	28	26	20

(b) Find the 1,2 and 3 fundamental harmonic of the Fourier series of f(x) given by the following table **(8)**

x	0	1	2	3	4	5
y	4	8	15	7	6	2

10 (a).Calculate the first two harmonic of Fourier series from the following data **(8)**

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0

(b).Find the Fourier series upto first harmonic **(8)**

T(sec)	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	2T
A(amp)	1.98	1.3	1.05	1.3	-8.8	-2.5	1.98

UNIT – II
FOURIER TRANSFORM

PART-A

1. State Fourier integral theorem

2.Show that f(x) = 1, 0<x<∞ cannot be represented by a Fourier Integral.

3. Find the Fourier transform of f(x) if

$$f(x)=\begin{cases}1 ; |x|<a\\ 0 ; |x|>a>0\end{cases}$$

4. State parsevals identity in fourier transforms.

5. Write the Fourier transform pair

6. Find Fourier sine transform of $\frac{1}{x}$

7. What is the Fourier cosine transform of a function.

8. Find the Fourier cosine transform of

$$\begin{cases}\text{Cosx if } 0<x<a\\ \end{cases}$$

f(x) =
0 if x ≥a

9. Find Fourier cosine transform of e^{-ax},a>0.
10.Find the Fourier sine transform of e^{-x}
11. Define Fourier sine transform and its inversion formula
12. If F(s) is the Fourier transform of f(x) , then show that the Fourier transform of e^{iax}f(x) is F(s+a).
13.Prove that F_c[f(x)cosax] = $\frac{1}{2}\left[F_c(s+a)+F_c(s-a)\right]$where F_c denotes the Fourier cosine transform f(x).
14. If F(s) if the complex Fourier transform of f(x) then find F[f(x-a)]
15.What is the fourier transform of f(x-a) if the fourier transform of f(x) is f(s).
16. State and prove first shifting theorem.
17. If F_c(s) is the Fourier cosine transform of f(x). Prove that the Fourier cosine transform of f(ax) is $\frac{1}{a}F_c\left[\frac{s}{a}\right]$
18. If F(s) is the Fourier transform of f(x) , then find the Fourier transform of f(x-a).

19. If F_s(s) is the fourier sine transform of f(x),show that

$$F_s(f(s)\cos ax)=\frac{1}{2}[F_s(s+a)+F_s(s-a)]$$

20. State the convolution theorem for Fourier transforms.

PART-B

1.(a)Find the Fourier cosine transform of e^{-4x}. Deduce that

$$\int_0^\infty \frac{\cos 2x}{x^2+16}dx=\frac{\pi}{8}e^{-8} \text{ and } \int_0^\infty \frac{x\sin 2x}{x^2+16}dx=\frac{\pi}{2}e^{-8}$$ **(8)**

(b)Find the Fourier transform of

$$f(x)=\begin{cases}1, & \text{for } |x|<1\\ 0 & \text{otherwise}\end{cases}$$

Hence prove that $\int_0^\infty \frac{\sin x}{x}dx=\int_0^\infty \frac{\sin^2 x}{x^2}dx=\frac{\pi}{2}$ **(8)**

2.(a)Find the Fourier Sine transform of

$$f(x)=\begin{cases}\sin x, & 0<x<\pi\\ \mathbf{0} & \pi<x<\infty\end{cases}$$ **(8)**

(b)Prove that $e^{\frac{-x^2}{2}}$ is self reciprocal under Fourier Cosine transform. **(8)**

3. (a)Find the Fourier transform of $e^{-a|x|}$, a>0. Hence deduce that **(8)**

$$F\left(xe^{-a|x|}\right)=i\sqrt{\frac{2}{\pi}}\frac{2as}{\left(a^2+s^2\right)^2}$$

(b)Solve for f(x) from the integral equation $\int_0^\infty f(x)\cos\alpha x dx = e^{-\alpha}$ **(8)**

4.(a)F ind the Fourier sine transform of e^{-ax}, a>0 and hence deduce the inversion formula **(8)**

(b)Find the fourier cosine transform of f(x)= $\begin{cases} 1-x, 0\le x\le 1 \\ 0, otherwise \end{cases}$

Hence show that $\int_0^\infty \frac{\sin x}{x}dx = \int_0^\infty \frac{\sin^2 x}{x^2}dx = \frac{\pi}{2}$ **(8)**

5. (a)Find the Fourier transform of f(x) given by f(x) = $1-x^2$ for|x| ≤1
= 0 for |x| ≥1

Hence evaluate $\int_0^\infty \left[\frac{\sin x - x\cos x}{x^3}\right]\cos\left(\frac{x}{2}\right)dx$ **(8)**

(b)Find the Fourier transform of f(x) if

$$f(x) = \begin{cases} 1-|x| & \text{for } |x|<1 \\ 0 & \text{for } |x|>1 \end{cases}$$ **(8)**

6 (a) If F[f(x)] = $\overline{f}(s)$ $prove that F[f(ax)] = \frac{1}{|a|}\overline{f}\left(\frac{s}{a}\right)$ **(8)**

(b)Find the Fourier sine transform of $\frac{e^{-ax}}{x}$, where a>0. **(8)**

7.(a) Find Fourier Cosine transform of $e^{-a^2x^2}$ and hence find Fourier sine transform of x$e^{-a^2x^2}$ **(8)**

(b) Use transform method to evaluate $\int_0^\infty \frac{dx}{\left(x^2+1\right)\left(x^2+4\right)}$ **(8)**

8.(a)Find the Fourier sine transform of f(x)=$1-x^2$, 0<x<1
= 0, otherwise

Hence prove that $\int_0^\infty \left[\frac{\sin x - x\cos x}{x^3}\right]\cos\left(\frac{x}{2}\right)dx = \frac{3\pi}{16}$. **(8)**

(b). Find the Fourier transform of

$$f(x) = \begin{cases} x & \text{for } |x| \le a \\ 0 & \text{for } |x| > a \end{cases}$$ **(8)**

9.(a) Find Fourier cosine transform e^{-x^2} **(8)**

(b) Find the Fourier sine transform of $e^{-|x|}$.

Hence show that $\int_0^{\infty}\left[\frac{x\sin x}{(1+x)^3}\right]dx = \frac{\pi}{2}e^{-a}, m > 0$ **(8)**

10.(a)Find Fourier sine transform and cosine transform of e^{-x} and hence find the Fourier sine transform of $\frac{x}{(1+x)^2}$ and Fourier cosine transform of $\frac{1}{(1+x)^2}$ **(8)**

(b).Find the Fourier sine transform of $xe^{-x^2/2}$ **(8)**

UNIT III
PARTIAL DIFFERENTIAL EQUATIONS

PART-A

1. Form the PDE by eliminating a and b from $z = (x^2+a^2)(y^2+b^2)$.
2. Find the PDE of the family of spheres having their centres on the line x=y=z.
3. Form a PDE by eliminating the function from the relation $z = f(\frac{x}{y})$.
4. Form a PDE of eliminating the arbitrary function Φ from Φ(x-y, x+y+z)=0.
5. Find the complete integral of q = 2px.
6. Form the p.d.e with $z = e^y f(x + y)$ as solution.
7. Form the p.d.e from $z=ax^3+by^3$
8. Define complete solution.
9. Define general solution.
10. Define particular solution of a p.d.e
11. Find the complete integral of p+q = pq
12. Solve $(D^2 – DD'-2D'^2)z = 0$
13. Solve $(4D^2+12DD'+9D'^2) z = 0$
14. Find the particular integral of $(D^2 +3DD'+2D'^2) z = x+y$
15. Find the particular integral of $(D^2 -3DD'-2D'^2) z = \cos(x+3y)$
16. Solve $(D_x + D_y)^2 = e^{x+y}$
17. Form the p.d.e by eliminating λ and μ from $(x-\lambda)^2+(y-\mu)^2+z^2=1$
18. Find the solution of p√x+q√y=√z
19. Find the complete integral of p+q=x+y
20. Solve $\frac{\partial^2 u}{\partial x\partial t} = e^{-t}\cos x$ if u = 0 when t = 0 and $\frac{\partial u}{\partial t} = 0$ at x = 0.

PART-B

1. (a) Form the PDE by eliminating the arbitrary function from the relation $f(xy+z^2, x+y+z) = 0$. **(8)**
   (b) Form the PDE by eliminating the arbitrary functions f and g in $z = f(x^3+2y) + g(x^3-2y)$. **(8)**
2. (a) Solve (3z-4y)p+(4x-2z)q=2y-3x
   (b) Solve $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$
3. (a) Solve x(y-z)p+y(z-x)q=z(x-y) **(8)**
   (b) Solve $x(y^2-z^2)p + y(z^2-x^2)q – z(x^2-y^2) = 0$. **(8)**
4. (a) Solve $(x^2 - y^2 - z^2)p + 2xyq = 2xz$ **(8)**

(b) Solve $(x^2-yz)p + (y^2-zx)q = (z^2-xy)$ **(8)**

5. (a) Solve $(y+z)p + (z+x)q = x+y$ **(8)**
   (b) Obtain the complete and general integral of $p^2+q^2 = x + y$ **(8)**
6. (a) Find the singular solution of $z = px + qy + \sqrt{p^2+q^2+16}$. **(8)**
   (b) Find the complete solution of $9(p^2z + q^2) = 4$. **(8)**
7. (a) Solve $z^2(p^2+q^2) = x^2 + y^2$. **(8)**
   (b) Solve $p^2 + q^2 = x^2 + y^2$. **(8)**
8. (a) Solve $(D^2-DD'-30D'^2)z = xy + e^{6x+y}$. **(8)**
   (b) Solve $(D^3-7DD'^2-6D'^3)z = \cos(x+2y) + x$. **(8)**
9. (a) Solve $(D^3+D^2D'-DD'^2-D'^3)z = e^x\cos 2y$. **(8)**
   (b) Solve $(D^2-DD'+D'-1)z = \cos(x+2y)$. **(8)**
10. (a) Solve $(D^2 - 2DD')z = x^3y + e^{2x}$ **(8)**
    (b) $(D^2+3DD'+2D'^2)z = \sin(2x+y) + x + y$ **(8)**

UNIT – IV
APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS

PART-A

1. Find the type of the pde: $4u_{xx}+4u_{xy}+u_{yy}+2u_x-u_y=0$
2. How many conditions needed to solve the one dimensional heat equation?
3. Write the one dimensional wave equation with initial and boundary conditions in which the initial position of the string is f(x) and the initial velocity imparted at each point x is g(x).
4. In steady state conditions derive the solution of one dimensional heat flow equation.
5. What are the various Solutions of $\frac{\partial^2 y}{\partial t^2}=a^2\frac{\partial^2 y}{\partial x^2}$.
6. What is the basic difference between the solution of one dimensional wave equation and one dimensional heat equation.
7. A string is stretched and fastened to two points l apart. Motion is started by displacing the string into the form $y=y_0\sin \pi x/l$ which it is released at time t=0. Formulate this problem as the boundary value problem.
8. What is the constant a^2 in the wave equation $U_{tt} = a^2u_{xx}$ or In the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$ what does c^2 stand for ?
9. State the suitable Solution of the one dimensional heat equation $\frac{\partial u}{\partial t} = a^2\frac{\partial^2 u}{\partial x^2}$
10. State the governing equation for one dimensional heat equation and necessary conditions to solve the problem
11. Write all variable separable Solutions of the one dimensional heat equation $u_t=\alpha^2u_{xx}$
12. State any two laws which are assumed to derive one dimension heat equation.
13. A rod of length 20cm whose one end is kept at 30^0C and the other end is kept at 70^0C is maintained so until steady state prevails. Find the steady state temperature.
14. A bar of length 50cms has its ends kept at 20^0C and 100^0C until steady state

conditions prevail. Find the temperature any point of the bar.

15. A rod 30cm long has its ends A and B kept at 20^0C and 80^0C respectively until steady state conditions prevail. Find the steady state temperature in the rod.
16. State two-dimensional Laplace equation.
17. Write down the periodic solutions of the Laplace equation in Cartesian coordinates
18. Find the steady state temperature distribution in a rod of length 10 cm whose ends x=0 and x=10 are kept at 20^0C and 50^0C respectively.
19. A square plate has its faces and the edge y=0 insulated. It's edges x=0 and x=π are kept at zero temperature and its fourth edge y =π is kept at temperature $\pi x - x^2$ .Write the boundary conditions alone.
20. An infinitely long rectangular plate with insulated surface is 10 cm wide. The two long edges and one short edge are kept at zero temperature while the other short edge x=0 is kept at temperature given by u = 20y , 0≤y≤5
    = 20(10-y),5≤y≤10.

Give the boundary conditions.

PART-B

1. A string is stretched and fastened to 2 points x=0 and x=*l* . motion is started by displacing the string into the form $y=k(lx-x^2)$ from which it is released at time t=0. Find the displacement of any point on the string at a distance of x from one end at time t. **(16)**
2. A string of length 2*l* is fastened at both ends . the mid point of the string is taken to a height b and then released from rest in that position. Find the displacement. **(16)**
3. A tightly stretched string with fixed end points x=0 and x=L, is initially in its equilibrium position. If it is set vibrating giving each velocity 3x(L-x), find the displacement **(16)**
4. A tightly stretched flexible string has its ends fixed at x=0 and x=l . At time t=0 , the string is given a shape defined by $f(x)=kx^2(l-x)$, where k is a constant , and then released from rest. Find the displacement of any point x of the string at any timet>0. **(16)**
5. The points of trisection of a string are pulled aside through a distance b on opposite sides of the position of equilibrium and the string is released from rest. Find an expression for the displacement. **(16)**
6. If a string of length *l* is initially at rest in its equilibrium position and each of its points is given a velocity v such that

   V= kx for 0<x<l/2

   = k(*l* - x) for l/2 <x <l

   show that the displacement at any time t is given by

   $$y(x,t)= \frac{4l^2c}{\pi^3 a}\left[\sin\frac{\pi x}{l}\sin\frac{\pi at}{l}-\frac{1}{3^3}\sin\frac{3\pi x}{l}\sin\frac{3\pi at}{l}+....\right]$$ **(16)**
7. A rod of length l has its end A and B kept at 0^0C and 100^0C respectively until steady state conditions prevail. If the temperature at B is reduced suddenly to 75^0C and at the same time the temperature at A raised to 25^0C find the temperature u(x,t) at a distance x from A and at time t. **(16)**
8. The ends A and B of a rod lcm long have the temperatures 40c and 90c until steady state prevails. The temperature at A is suddenly raised to 90c and at the same time

that at B is lowered to 40c . Find the temperature distribution in the rod at time t. Also show that the temperature at the mid point of the rod remains unaltered for all time, regarless of the material of the rod. **(16)**

9. The ends A and B of a rod *30* c.m. long have their temperatures kept at $20^{\cdot}$c and $80^{\cdot}$c, until steady state conditions prevail. The temperature of the end B is suddenly reduced to $60^{\cdot}$c and that of A is increased to $40^{\cdot}$c . Find the temperature distribution in the rod after time t. **(16)**
10. An insulated rod of length l its ends A and B are maintained at 0^0C and 100^0C respectively until steady state conditions prevail. If B is suddenly reduced to 0^0C and maintained so, find the temperature at a distance x from A at time t. **(16)**
11. A bar of 10cm long, with insulated sides has its ends A and B maintained at temperatures 50^0C and 100^0C respectively, until steady-state conditions prevail.The temperature at A is suddenly raised to 90^0C and at B is lowered to 60^0C . Find the temperature distribution in the bar thereafter. **(16)**
12. An infinitely long uniform plate is bounded by two parallel edges and an end at right angle to them. The breadth of this edge x=0 is π, this end is maintained at temperature as $u=K(\pi y-y^2)$ at all points while the other edges are at zero temperature. Find the temperature u(x,y) at any point of the plate in the steady state. **(16)**
13. A rectangular plate with insulated surface is 10cm wide and so long compared to its width that it may be considered infinite in length without introducing appreciable error. The temperature at short edge y=0 is given by

$$U = \begin{cases} 20x & \text{for } o<x<5 \\ 20(10-x) & \text{for } 5<x<10 \end{cases}$$

and all the other three edges are kept at 0c . Find the steady state temperature at any point in the plate. **(16)**
14. A rectangular plate with insulated surfaces is 'a' cm wide and so long compared to its width that it may be considered infinite in length, x=a and the short edge at infinity are kept at temperature $0^{\cdot}$c, while the other short edge y=0 is kept at temperature $u_0 \sin^3\left(\frac{\pi x}{a}\right)$, , find the steady state temperature at any point (x,y) of the plate. **(16)**
15. Find the steady state temperature distribution in a rectangular plate of sides a and b insulated at the lateral surface and satisfying the boundary conditions u(0,y)=u(a,y)=0 for o<y<b, u(x,b)=0 and u(x,0)=x(a-x) for 0<x<a. **(16)**
16. A square plate is bounded by the lines x=0,y=0,x=l and y=l and its faces are insulated. The temperature along the upper horizontal edge is given by $u(x, l)=lx-x^2$ for 0<x<l, while the other three edges are maintained at 0^oC. Find the steady-state temperature distribution in the plate. **(16)**

UNIT – V
Z- TRANSFORM

PART-A

1. Define Z- Transforms.
2. Find Z[e^{at+b}]
3. Prove that Z$[a^n]$ = $\frac{z}{z-a}$ and deduce that z[1]
4. Find the Z$\left[\frac{1}{n(n+1)}\right]$
5. Find Z$[\cos n\theta]$ and Z$[\sin n\theta]$
6. Find Z$\left[\frac{a^n}{n!}\right]$
7. Prove that Z(n) = $\frac{z}{(z-1)^2}$
8. Find Z(n^2)
9. Find the Z- transform of (n+1)(n+2)
10. Find Z$[e^t \sin 2t]$
11. Find Z(t)
12. Find $Z(\frac{1}{n})$
13. Evaluate $Z^{-1}\left(\frac{z}{z^2+7z+10}\right)$
14. Evaluate $Z^{-1}\left(\frac{z^2}{(z-a)(z-b)}\right)$
15. Find Z$[f(n+1)]$ = Z F(z) – z f(0)
16. Find the Z-transform of (n_{c_k})
17. Prove that Z$[nf(n)]$ = -z $\frac{dF(z)}{dz}$
18. State the initial value theorem in Z-transforms If Z[f(t)]=F(z),then f(0)= $\lim_{z\to\infty} F(z)$
19. Form the difference equation from $y_n = a + b3^n$
20. Form the difference equation by eliminating arbitrary constants from $U_n = \alpha 2^{n+1}$

PART-B

1. (a) Prove that $Z\left[\frac{1}{(n+1)}\right] = z \log\left[\frac{z}{z-1}\right]$ **(8)**

(b)Find $Z\left[\frac{1}{(n+1)(n+2)}\right]$ by using method of partial fraction. **(8)**

2. (a)Find $Z\left[a^n r^n \cos n\theta\right]$ and $Z\left[a^n r^n \sin n\theta\right]$ **(8)**

(b) Find the Z transform of $\frac{1}{n+1}$ and $2^n \cos\frac{n\pi}{2}$. **(8)**

3. (a) Find $Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right]$ by using Convolution theorem. **(8)**

(b)Using Convolution theorem evaluate $Z^{-1}\left[\frac{8z^2}{(2z-1)(4z+1)}\right]$ **(8)**

4. (a)Find $Z^{-1}\left[\frac{9z^3}{(3z-1)^2(z-2)}\right]$ by using residue method. **(8)**

(b)Find the inverse z-transform of $\frac{z(z+1)}{(z-1)^3}$ using the method of residues. **(8)**

5. (a) Find $Z^{-1}\left[\frac{z(z^2-z+2)}{(z+1)(z-1)^2}\right]$ by using method of partial fraction. **(8)**

(b) Find $Z^{-1}\left[\frac{z^2-3z}{(z+2)(z-5)}\right]$ by residue method. **(8)**

6. (a)Find $Z^{-1}\left[\frac{2z^2-10z+13}{(z-3)^2(z-2)}\right]$ when 2<|z|<3 **(8)**

(b)Find $Z\left[t^k\right]$ deduce that $Z\left[t^2\right]$. **(8)**

7 (a)State and Prove Convolution theorem on Z-transforms **(8)**

(b)State and Prove initial value and Final value theorem. **(8)**

8 (a)Derive the difference equation from $y_n=(A+Bn)(-3)^n$ **(8)**

(b)Derive the difference equation from $u^n=A2^n+Bn$ **(8)**

9 (a)Solve $y_{n+2}-4y_{n+1}+3y_n=0$ given $y_0=2$ and $y_1=4$. **(8)**

(b)Using Z- Transform solve the equation $u_{n+2}+3u_{n+1}+2u_n=0$ given u(0) = 1 and u(1) =2. **(8)**

10 (a)Using Z- Transform solve the equation $u_{n+2}-5u_{n+1}+6u_n=4^n$ given u(0) = 0 and u(1) =1. **(8)**

(b)Using Z- Transform solve the equation $y_{n+2}+4y_{n+1}-5y_n=24n-8$ given y(0) = 3 and y(1) = -5. **(8)**