KINGS

COLLEGE OF ENGINEERING
Punalkulam

DEPARTMENT OF MATHEMATICS
ACADEMIC YEAR – 2011 – 12(ODD SEMESTER)

QUESTION BANK

SUBJECT CODE/NAME: MG1402 - OPERATIONS RESEARCH
YEAR/SEM: IV/VII **BRANCH: EEE**

UNIT-I
LINEAR PROGRAMMING(LP)

PART-A

1. What is operations research?
2. What are the phases of an operations research study?
3. Define a feasible solution.
4. Define optimal solution.
5. What is the difference between feasible solution and basic feasible solution?
6. Define unbounded solution.
7. What are the two forms of a LPP?
8. What do you mean by standard form of LPP?
9. What do you mean by canonical form of LPP?
10. What are the limitations of LPP?
11. What are the slack and surplus variables?
12. what is meant by decision variable?
13. Define artificial variable.
14. What are the methods used to solve an LPP involving artificial variables?
15. What is degeneracy?

PART-B

1. (a) Explain the scope of OR. **(8)**
   (b) List the phases of OR and explain them. **(8)**
2. (a) A company manufactures two products A and B. Each unit of B takes twice as long to produce as one unit of A and if the company were to produce only A it would have time to produce 2000 units per day. The availability of the raw material is sufficient to produce 1500 units per day of both a and B combined. Product B recuiring a special Ingredient only 600units can be made per day. If A fatches a profit of Rs.2 per unit and B a profit of Rs.4 per unit, Formulate the optimum product min. **(8)**

(b) A paper mill produces 2 grades of paper namely X and Y. Because of raw material restrictions, it cannot produce more than 400 tonnes of grade X and 300 tonnes of grade Y in a week. There are 160 production hours in a week. It requires 0.2 and 0.4 hours to produce a ton of products X and Y respectively with corresponding profits of Rs.200 and Rs. 500 per ton. Formulate the above as a LPP to maximize profit and find the optimum product mix. **(8)**

3 (a) A company produces 2 types of hats. Every hat A require twice as much labour time as the second hat be. If the company produces only hat B then it can produce a total of 500 hats a day. The market limits daily sales of the hat A and hat B to 150 and 250 hats. The profits on hat A and B are Rs.8 and Rs.5 respectively. Solve graphically to get the optimal solution. **(8)**

(b) Use Graphical method to solve the following LP problem

Maximize $Z= 15x_1+10x_2$

Subject to the constraints: $4x_1+6x_2 \leq 360$

$3x_1+0x_2 \leq 180$

$0x_1+5x_2 \leq 200$; $x_1, x_2 \geq 0$ **(8)**

4 Use simplex method to solve the following LPP

Maximize $Z = 4x_1+10x_2$

Subject to the constraints

$2x_1+x_2 \leq 50,$

$2x_1+5x_2 \leq 100,$

$2x_1+3x_2 \leq 90,$

$x_1, x_2 \geq 0$ **(16)**

5. Solve the following problem by simplex method

Minimize $Z = x_1-3x_2+2x_3$

Subject to the constraints

$3x_1-x_2+2x_3 \leq 7,$

$-2x_1+4x_2 \leq 12,$

$-4x_1+3x_2+8x_3 \leq 10,$

$x_1, x_2, x_3 \geq 0$ **(16)**

6. Use Big M method to

Maximize $Z = 3x_1+2x_2$

Subject to the constraints

$2x_1+x_2 \leq 2,$

$3x_1+4x_2 \geq 12,$

$x_1, x_2 \geq 0$ **(16)**

7. Use Penalty method

Maximize $Z = 6x_1+4x_2$

Subject to the constraints

$2x_1+3x_2 \leq 30,$

$3x_1+2x_2 \leq 24$

$x_1+x_2 \geq 3,$

$x_1, x_2 \geq 0$ **(16)**

8. Use Two Phase Simplex method to

Maximize $Z = 5x_1-4x_2+3x_3$

Subject to the constraints

$2x_1+x_2-6x_3 = 20,$

$6x_1+5x_2+10x_3 \le 76,$

$8x_1-3x_2+6x_3 \le 50,$

$x_1, x_2, x_3 \ge 0$ **(16)**

9. Use Two Phase Simplex Method to

Maximize $Z = -4x_1-3x_2-9x_3$

Subject to the constraints

$2x_1+4x_2+6x_3 \ge 15,$

$6x_1+x_2+6x_3 \ge 12,$

$x_1, x_2, x_3 \ge 0$ **(16)**

10. Use Artificial variable technique to solve the following LP Problem"

Maximize $Z=x_1+2x_2+3x_3-x_4$

Subject to : $x_1+2x_2+3x_3=15$

$2x_1+x_2+5x_3=20$

$x_1+2x_2+x_3+x_4=10$

$x_1, x_2, x_3, x_4 \ge 0$ **(16)**

UNIT-II
DUALITY, TRANSPORTATION MODEL AND ASSIGNMENT MODEL

PART-A

1. Define dual of LPP.
2. What are the importance of the duality concept?
3. State the optimality condition in dual simplex method.
4. What is the difference between regular simplex method and dual simplex method?
5. What do you understand by transportation problem?
6. List any three approaches used with T.P for determining the starting solution.
7. What do you mean by degeneracy in a T.P?
8. What do you mean by an unbalanced T.P?
9. How do you convert the unbalanced T.P into a balanced one?
10. What is an assignment problem?
11. List the various methods are used to solve the assignment problems?
12. What do you mean by an an unbalanced assignment problem?
13. State the difference between the T.P and A.P.
14. What is the objective of the travelling salesman problem?
15. How do you convert the maximization assignment problem into a minimization one?
16. Explain how the profit maximization transportation problem can be converted to an equivalent cost minimization transportation problem

PART-B

1. (a) Obtain the dual of the following primal problem

Minimize $z=3x_1-2x_2+x_3$

Subject to : $2x_1-3x_2+x_3 \le 5$

$4x_1-2x_2 \ge 9$

$-8x_1+4x_2+3x_3=8$

$x_1, x_2 \ge 0$, x_3 is unrestricted. **(8)**

(b) Explain the rules for constructing the dual from primal.

2. Use dual simplex method to

Maximize $Z = -3x_1-2x_2$

Subject to : $x_1+x_2 \ge 1$

$x_1+x_2 \le 7$

$x_1+2x_2 \ge 10$

$x_2 \le 3$

$x_1, x_2 \ge 0$ **(16)**

3. Solve by Dual simplex method the following LPP

Minimize $Z = 5x_1+6x_2$

Subject to the constraints

$x_1+x_2 \ge 2$,

$4x_1+x_2 \ge 4$

$x_1, x_2 \ge 0$ **(16)**

4. Use dual simplex method to solve the LPP

Maximize $Z = -2x_1-x_3$

Subject to the constraints

$x_1+x_2-x_3 \ge 5$,

$x_1-2x_2+4x_3 \ge 8$,

$x_1, x_2, x_3 \ge 0$ **(16)**

5. Find the non-degenerate basic feasible solution for the following transportation problem using:

a) North-west corner Rule

b) Least cost Method

c) Vogel's Approximation method.

	To				Supply
From	3	12	5	19	10
	10	20	5	7	20
	13	9	12	8	30
	4	5	7	9	40
	14	7	1	0	50
Demand	60	60	20	10	

(16)

6. Solve the following transportation problem starting with the initial solution obtained by VAM **(16)**

	D_1	D_2	D_3	D_4	Supply
O_1	2	2	2	1	3
O_2	10	8	5	4	7
O_3	7	6	6	8	5
Required	4	3	4	4	15

7. A company has 3 plants A,B, and C , three warehouses X,Y,Z. A number of units available at the plants is 60,70,80 and the demand at X,Y,Z are 50,80,80 respectively. The unit cost of the transportation is given in the following table

	X	Y	Z
A	8	7	3
B	3	8	9
C	11	3	5

Find the allocation so that the total transportation cost is minimum. **(16)**

8.(a) Obtain an initial basic feasible solution to the following TP using VAM **(8)**

	D_1	D_2	D_3	D_4	Supply
O_1	11	13	17	14	250
O_2	16	18	14	10	300
O_3	21	24	13	10	400
Demand	200	225	275	250	950

(b)A company has 4 machines to do 3 jobs. Each job can be assigned to one and only one machine. The cost of each job on each machine is given below. Determine the job assignments which will minimize the total cost. $\begin{pmatrix} 18 & 24 & 28 & 32 \\ 8 & 13 & 17 & 18 \\ 10 & 15 & 19 & 22 \end{pmatrix}$ **(8)**

UNIT-III
INTEGER DYNAMIC PROGRAMMING

PART-A

1. What do you mean by integer programming problem?
2. What are the applications of zero-one integer programming?
3. Define a mixed integer programming problem.
4. Differentiate between pure and mixed IPP.
5. What are the methods used in solving IPP
6. Explain Gomorian constraint (or) Fractional Cut constraint.
7. Where is branch and bound method used?
8. What is dynamic programming?
9. Define the terms in dynamic programming : stage, state ,state variables
10. Give a few applications of DPP.
11. State Bellman's principle of optimality.
12. What are the advantages of Dynamic programming?
13. Explain the importance of the L.P.P

PART B

1. Solve the following mixed integer programming problem by using Gomory's cutting plane method

Maximize $Z = x_1+x_2$

Subject to the constraints

$3x_1+2x_2 \leq 5,$

$x_2 \leq 2$

$x_1, x_2 \geq 0$, x1 is an integer **(16)**

2. Find an optimum integer solution to the following LPP

Maximize $Z = x_1+2x_2$

Subject to the constraints

$2x_2 \leq 7,$

$x_1+x_2 \leq 7$

$2x_2 \leq 11$

$x_1, x_2 \geq 0$, x_1, x_2 are integers **(16)**

3. Solve the following integer programming problem

Maximize $Z = 2x_1+20 x_2-10x_3$

Subject to the constraints

$2x_1+20x_2+4x_3 \leq 15,$
$6x_1+20x_2+4x_3 = 20,$
$x_1, x_2, x_3 \geq 0$ and are integers **(16)**

4. Solve the following all integer programming problem using the Branch and bound method.

   Minimize $Z = 3x_1+2.5x_2$
   Subject to the constraints
   $x_1+2x_2 \geq 20,$
   $3x_1+2x_2 \geq 50$
   and x_1, x_2 are nonnegative integers **(16)**

5. Use Branch and Bound technique to solve the following

   Maximize $Z = x_1+4x_2$
   Subject to the constraints
   $2x_1+4x_2 \leq 7,$
   $5x_1+3x_2 \leq 15$
   $x_1, x_2 \geq 0$ and are integers **(16)**

6. A student has to take examination in three courses X,Y,Z. He has three days available for study. He feels it would be best to devote a whole day to study the same course, so that he may study a course for one day,two days or three days or not at all. His estimates of grades he may get by studying are as follows

Study days/ course	X	Y	Z
0	1	2	1
1	2	2	2
2	2	4	4
3	4	5	4

How should he plan to study so that he maximizes the sum of his grades? **(16)**

7. A student has to take examinations is three courses A,B and C. He has three days available for study. He feels is would be best to denote a whole day to the study of the same course. So that he may study a course for one day, two days or three days or not at all. His estimates of grades he may get by study are as follows

Course / study days	A	B	C
0	0	1	0
1	1	1	1
2	1	3	3
3	3	4	3

How should he plan to study so that he maximizes the sum of his grades.

8. The owner of a chain of four grocery stores has purchased six crates of fresh strawberries. The estimated probability distribution of potential sales of the strawberries before spoilage differ among four stores. The following table gives the estimated total expected profit at each store when various number of crates are allocated to it. For administrative reasons, the owner does not wish to split crates between stores. However, he is willing to distribute zero crates to any of his stores. Find the allocation of six crates to four stores so as to maximize the expected profit

Number of Crates	Stores			
	1	2	3	4
0	0	0	0	0
1	4	2	6	2
2	6	4	8	3
3	7	6	8	4
4	7	8	8	4
5	7	9	8	4
6	7	10	8	4

(16)

9. Use dynamic programming to solve the following LPP

Maximize $Z = x_1+9x_2$

Subject to the constraints

$2x_1+x_2 \leq 25$

$x_2 \leq 11,$

$x_1, x_2 \geq 0$ **(16)**

UNIT-IV

PROJECT MANAGEMENT AND THEORY OF GAMES

PART-A

1. What is a network?
2. What is merge event?
3. What are the three types of float?
4. What is total float?
5. Define critical activity and critical path.
6. Distinguish between PERT and CPM

7. Define the cost time slope of an activity.
8. Define crash time and crash cost
9. Define a game.
10. Define strategy.
11. Define two person zero sum game.
12. Define pure and mixed strategies.
13. Define saddle point
14. When do players apply mixed strategies?

PART B

1. A project schedule has the following characteristics

Activity	1-2	1-3	2-4	3-4	3-5	4-9	5-6	5-7
Time(days)	4	1	1	1	6	5	4	8
Activity	6-8	7-8	8-10	9-10				
Time(days)	1	2	5	7				

From the above table

(1) Construct a network diagram
(2) Compute the earliest event time and latest event time
(3) Determine the critical path and total project duration
(4) Compute total float, free float for each activity. **(16)**

2. A project has the following activities and other characteristics:

	Estimated Duration (in weeks)		
Activity (i-j)	Optimistic	Most likely	Pessimistic
1-2	1	1	7
1-3	1	4	7
1-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-6	2	5	8
5-6	3	6	15

(i) what is the expected project length?
(ii) What is the probability that the project will be completed no more than 4 weeks later than expected time? **(16**

3. Listed in the table are the activities and sequencing requirements necessary for the completion of research report.

Activity	A	B	C	D	E	F	G	H	I	J	K	L	M

Duration	4	2	1	12	14	2	3	2	4	3	4	2	2
Immediate Predecessors	E	A	B	K	-	E	F	F	F	I,L	C,G,H	D	I,L

Find the critical path

(i) Find the total float and the free float for each non-critical activity **(16)**

4. A small maintenance project consists of the following jobs whose precedence relationships is given below

Job	1-2	1-3	2-3	2-5	3-4	3-6	4-5	4-6	5-6	6-7
Duration (days)	15	15	3	5	8	12	1	14	3	14

(i) Draw an arrow diagram representing the project

(ii) Find the total float for each activity

(iii) Find the critical path and the total project duration **(16)**

5.(a)Explain in detail about various phases of project management. **(6)**

(b). Calculate the earliest start, earliest finish, latest start and latest finish of each activity of the project given below and determine the critical path of the project

Activity :	1-2	1-3	1-5	2-3	2-4	3-4	3-5	3-6	4-6	5-6
Duration : (in weeks)	8	7	12	4	10	3	5	10	7	4

(10)

6.The following table shows the jobs of a network along with their time estimates

Job	1-2	1-6	2-3	2-4	3-5	4-5	6-7	5-8	7-8
a(days)	1	2	2	2	7	5	5	3	8
m(days)	7	5	14	5	10	5	8	3	17
b(days)	13	14	26	8	19	17	29	9	32

Draw the project network and find the probability that the project is completed in 40 days. **(16)**

7. The following table shows the jobs of a network along with their time estimates

Job	1-2	1-6	2-3	2-4	3-5	4-5	5-8	6-7	7-8
a(days)	3	2	6	2	5	3	1	3	4
m(days)	6	5	12	5	11	6	4	9	19
b(days)	15	14	30	8	17	15	7	27	28

(i)Draw the project network.

(ii)Find the critical path

(iii) Find the probability that the project is completed in 31 days. **(16)**

8. A small project is composed of seven activities whose time estimates are listed in the table as follows:

Estimated duration (weeks)			
Activity	Optimistic	Most likely	Pessimistic
1-2	1	1	7
1-3	1	4	7
2-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-6	2	5	8
5-6	3	6	15

Draw the project network

(i) Calculate the variance and standard deviation of project length

(ii) What is the probability that the project will be completed 4 weeks earlier than expected? **(16)**

9. A Project has the following activities and other characteristics:

Estimated Duration (in weeks)

Activity (i-j)	Optimistic	Most likely	Pessimistic
1-2	1	1	7
1-3	1	4	7
1-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-6	2	5	8
5-6	3	6	15

(i) What is the expected project length?

(ii) What is the probability that the project will be completed no more than 4 weeks later than expected time?

10. Use the graphical method in solving the following game and find the value of the game

Player A	Player B B_1	B_2	B_3	B_4
A_1	2	2	3	-2
A_2	4	3	2	6

UNIT V
INVENTORY CONTROL AND QUEUING

PART A

1. What is meant by inventory?
2. Mention the various types of inventory.
3. What are the different costs that are involved in the inventory problem?
4. Define holding cost and setup cost
5. Briefly explain probabilistic inventory model.
6. Distinguish between deterministic model and probabilistic model
7. Define buffer stock or safety stock.
8. Define a queue
9. What are the basic characteristics of a queuing system?
10. Define transient and steady state.
11. Explain Kendall's notation.
12. Write Little' formula
13. Define the following (1) Balking (2) Reneging (3) Jockeying
14. List the characteristic of a queueing system
15. Explain the queue discipline and its various forms:

PART – B

1. (a) A company purchases 9000 parts of a machine for its annual requirement, ordering one month's usage at a time. Each part cost Rs.20. The ordering cost per order is Rs.15 and the carrying charges are 15% of the average inventory per year. You have been asked to suggest a more economical purchasing policy for the company. What advice would you offer and how much would it save the company per year? **(8)**

   (b) The annual requirement for a product is 3000 units. The ordering cost is Rs. 100 per order. The cost per unit is Rs.10. The carrying cost per unit per year is 30% of the unit cost.

   (a) Find the EOQ

   (b) By using better organizational methods the ordering cost per order is brought to Rs.80 per order, but the same quantity as determined above were ordered.

   (c) If a new EOQ is found by using the ordered cost as Rs.80, what would be further savings in cost? **(8)**

2. (a) The demand for an item is 18000 units per year. The holding cost is Rs1.20 per unit time and the cost of shortage is Rs.5.00. The production cost is Rs.400.00. Assuming that replacement rate is instantaneous determine the optimum order quantity. **(8)**

   (b) The demand for an item is 12000 per year and the shortage is allowed. If the unit cost is Rs.15 and the holding cost is Rs.20 per year per unit determine the optimum total yearly cost. The cost of placing one order is Rs.6000 and the cost of one shortage is Rs.100 per year. **(8)**

3 (a) The annual consumption of an item is 2000 items. The ordering cost is Rs.100 per order. The carrying cost is Rs0.80 per unit per year. Assuming working days as 200 lead time 20 days, and safety stock 100 units.

Calculate (a)EOQ (b) the number of orders per year. (c) Reorder level. (d) the total annual ordering and carrying cost. **(8)**

(b) The following table gives the distributions for lead time and daily demand during the lead time. Determine the buffer stock. **(8)**

L (days)	3	4	5	6	7	8	9	10
Frequency(f)	2	3	4	4	2	2	2	1
Demand	0	1	2	3	4	5	6	7
Frequency	2	4	5	5	4	2	1	2

4. Arrivals of a telephone booth are considered to be poisson with an average time of 10 min. between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 3 min.
(i) What is the probability that a person arriving at the booth will have to wait?
(ii) The telephone department will install a second booth, when convinced that an arrival has to wait on the average for at least 3 min. for phone. By how much the flow of arrivals should increase in order to justify a second booth?
(iii) What is the average length of the queue that forms time to time?
(iv) What is the probability that it will take him more than 10 minutes altogether to wait for the phone and complete his call? **(16)**

5. (i) Write the steady-state equation for the model (M/M/C):(FIFO/∞/∞)
(ii) Obtain the expected waiting time of a customer in the queue of the above model.
(iii) In the above model λ=10/hour, μ=3/hour C=4, what is the probability that a customer has to wait before he gets service? **(16)**

6. Customers arrive at a one window drive in bank according to Poisson distribution with mean 10 per hour. Service time per customer is exponential with mean 5 minutes. The space in front of the window including that for the serviced car can accommodate a maximum of 3 cars. Others can wait outside this space?
(i) What is the probability that an arriving customer can drive directly to the space in front of the window?
(ii) What is the probability that an arriving customer will have to wait outside the indicated space?
(iii) How long is an arriving customer expected to wait before starting service. **(16)**

7. (a) In a supermarket, the average arrival rate of customer is 10 every 30 minutes following Poisson process. The average time taken b y a cashier to list and calculate the customer's purchase is 2.5 minutes following exponential distribution. What is the probability that the queue length exceeds 6?
What is the expected time spent by a customer in the system? **(8)**

(b) In a public telephone booth the arrivals are on the average 15 per hour. A call on the average takes 3 minutes. If there is just one phone, find (i) the expected number of callers in the booth at any time (ii) the proportion of the time the booth is expected to be idle? **(8)**

8. (a) In a railway marshalling yard, goods train arrive at the rate of 30 trains per day. Assume that the inter arrival time follows an exponential distribution and the service time is also to be assumed as exponential with mean of 36 minutes. Calculate
   (i) The probability that the yard is empty.
   (ii) The average queue length assuming that the line capacity of the yard is 9 trains. **(10)**

   (b) A car park contains 5 cars. The arrival of cars is Poisson at a mean rate of 10 per hour. The length of time each car spends in the car park is exponential distribution with mean of 5 hours. How many cars are in the park on the average? **(6)**

9. A super market has two girls ringing up sales at the counters. If the service time for each customer is exponential with mean 4 minutes and if people arrive in a Poisson fashion at the counter at the rate of 10 per hour, then calculate
   (a) The probability of having to wait for service.
   (b) the expected percentage of idle time for each girl.
   (c) If a customer has to wait find the expected length of his waiting time. **(16)**

10. (a) A petrol pump station has 2 pumps. The service times follow the exponential distribution with a mean of 4 min and cars arrive for service in a poisson process at the rate of 10 cars per hour.Find the probability that a customer has to wait for service. What proportion of time the pumps remain idle? **(8)**

   (b) A barber shop has two barbers and three chairs for customers. Assume that the customers arrive in Poisson fashion at a rate of 5 per hour and that each barber services customers according to an exponential distribution with mean 15 minutes. Further, if a customer arrives and there are no empty chairs in the shop, he will leave. What is the expected number of customers in the shop? **(8)**

11**.** (a) Discuss briefly about the different types of inventory and various costs involved in inventory problems **(8)**

   (b)A company has a demand of 12,000 units/year for an item and it can produced 2000 such items per month. The cost of one setup is Rs.400 and the holding cost/unit/month is Rs. 0.15. Find the optimum lot size, max inventory, manufacturing time, total time. **(8)**