KINGS

COLLEGE OF ENGINEERING

DEPARTMENT OF MATHEMATICS
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QUESTION BANK

SUBJECT NAME:PROBABILITY AND QUEUEING THEORY
YEAR/SEM: II / IV

UNIT-I
RANDOM VARIABLES
PART-A(2 Marks)

1. If X is a discrete random variable with probability distribution P(X=x)=kx,x=1,2,3,4 find P(2<X<4).
2. Find K,if the p.d.f of X is

X	-1	0	1	2	3
P(X=x)	2k	3k	4k	$6k^2$	$4k^2$

3.The p.d.f of a continuous random variable X is f(x)= $\begin{cases} k(3+2x), for 2 \le x \le 4 \\ 0, otherwise \end{cases}$ find the value of K.

4.The continuous random variable x has a probability density function f(x)=k(1+x), 2 ≤x≤5.Find P(X<4).

5. A random variable X has the p.d.f f(x) given by f(x)= $\begin{cases} cxe^{-x}, if x > 0 \\ 0, if x \le 0. \end{cases}$ find the value of and C.D.F of X.

6. A continuous random variable X has the p.d.f f(x) given by f(x)= $Ce^{-|x|}$, $-\infty < x < \infty$. Find the value of C.

7. Find the mean and variance of the distribution whose moment generating function is $(0.4e^t+0.6)^2$.

8. What is the moment generating function of a random variable?

9. Find the moment generating function of continuous probability distribution whose density is $2e^{-2x}$, x≥0

10. The first four moments of a distribution about X = 4 are 1,4,10 and 45 respectively. Show that the mean is 5 , variance is 3 , μ_3=0 and μ_4=26.

11. The mean and variance of a binomial variate X are 4 and 4/3 respectively. Find $P(X \ge 1)$

12. Find the MGF of binomial distribution.

13. If the independent random variables X , Y are binomially distributed respectively with n=3 , p=$\frac{1}{3}$ and n=5 , p=$\frac{1}{3}$, find P(X+Y≥1).

14. If 3% of the electric bulbs manufactured by a company are defective , find the probability that in a sample of 100 bulbs exactly 5 bulbs are defective. (e^{-3}=0.0498)

15. Write the moment generating function of a geometric distribution.
16. If X is uniformly distributed over (0,10),find the probability that 3<x<9.
17. A random variable X has a uniform distribution over (-3,3) compute P($|X-2|$<2).
18. Define exponential distribution.
19. Write down the moment generating function of the Gamma distribution?
20. What is the relationship between weibull and Exponential distribution?

PART – B(16 Marks)

1.a. A random variable X has the following probability distribution :

X = x	-2	-1	0	1	2	3
P(x)	0.1	k	0.2	2k	0.3	3k

Find k ,P(-2<x<2). **(8)**

b. A random variable X has the probability density function f(x) = 2x ,0<x<1
= 0 , otherwise

find (1) $P(X < \frac{1}{2})$ (2) $P(\frac{1}{4} < X < \frac{1}{2})$ (3) $P(X > \frac{3}{4} / X > \frac{1}{2})$ **(8)**

2.a. The distribution function of a random variable X is given by $F(x)=1-(1+x)e^{-x}, x\geq 0$. Find the density function, mean and variance of X. **(8)**

b. Find the MGF of the random variable X whose p.d.f. is

$$f(x)=\begin{cases} x, 0 \leq x < 1 \\ 2-x, 1 \leq x < 2 \\ 0, otherwise \end{cases}$$ **(8)**

3.a. If the moments of a random variable X are defined by $E[X^r] = 0.6$; r =1,2,3,….show that P(X=0) = 0.4 ,P(X=1)=0.6 , P(X≥2) = 0. **(8)**

b. A discrete random variable X has moment generating function

$M_X(t) = (\frac{1}{4}+\frac{3}{4}e^t)^5$ find E(X) , Var (X) and P(X = 2). **(8)**

4.a. The p.d.f of a continuous random variable X is f(x)=Kx(1-x), 0≤x≤1. Find (i) the value of K (ii) distribution function of X and (iii)r^{th} moment about origin. **(8)**

b. Out of 800 families with 4 children each , how many families would be expected to have (i) 2 boys and 2 girls (ii) atleast 1 boy (iii) atmost 2 girls (iv) children of both genders. Assume equal probabilities for boys and girls. **(8)**

5.a. A machine manufacturing screws is known to produce 5% defective. In a random sample of 15 screws , what is the probability that there are (i) exactly 3 defectives (ii) not more than 3 defectives. **(8)**

b. Find the mean and variance of the binomial distribution **(8)**

6.a. Find the M.G.F.of the binomial distribution and hence find its mean and variance **(8)**

b. Find the M.G.F. of Poisson distribution.Hence find mean and variance of the distribution. **(8)**

7.a. 3% of the electric light bulbs manufactured by a company are found to be defectives. In a sample of 100 such bulbs,find the probability that
(i) all are good
(ii) atleast 3 defectives
(iii) exactly 2 defectives

(iv) atmost 4 defectives **(8)**

b. Find the moment generating function of the random variable with the probability law $P(X=x)=q^{x-1}p$;x=1,2,…..Find the mean and variance. **(8)**

8.a. Define Geometric distribution.If the probability that an applicant for a driver's licence will pass the road test on any given trial is 0.8, what is the probability that he will finally pass the test;

(i) on the fourth trial

(ii) in fewer than 4 trials. **(8)**

b. Find the mean, variance and m.g.f. of a r.v uniformly distributed in the interval (a,b) **(8)**

9.a. A random variable X has an uniform distribution over the interval (-3,3). Compute

(i) P[X=2]

(ii) P[X<2]

(iii) P [$|X| < 2$]

(iv) P [$|X-2| < 2$]

(v)Find k such that P(X>k)=$\frac{1}{3}$ **(8)**

b. Obtain the M.G.F of exponential distribution. From the M.G.F.,find the mean and variance and r^{th} moment about origin. **(8)**

10.a. The time in minutes that girl speaks over phone is a r.v. X with pdf f(x)=A$e^{-\frac{x}{5}}$, x>0. Find the probability that she uses phone

(i) for atleast 5 minutes

(ii) for atmost 10 minutes

(iii) between 5 and 10 minutes. **(8)**

b. State and prove memoryless property of exponential distribution. **(8)**

11.a. Derive the MGF of Gamma distribution and find its mean and variance. **(8)**

b. If the life (in yrs) of a certain type of car has a weibull distribution with the parameter β = 2 ,find the value of te parameter α,given that the probability that parameter α = $\frac{1}{2}$, what is the probability the required time (a) exceeds 2 hours and (b) exceeds 5 hours. **(8)**

UNIT - II

TWO DIMENSIONAL RANDOM VARIABLES

PART-A(2 Marks)

1.The joint probability density function of the random variable (X,Y) is given by f(x,y)=Kxy$e^{-(x^2+y^2)}$ x>0, y>0. Find the value of K.

2.The joint p.d.f. of (X,Y) is f(x,y)=4xy, 0<x,y<1 and f(x,y)=0, otherwise, Find E(XY).

3. The joint p.d.f. of (X,Y) is f(x,y)=$\begin{cases} 4xy, 0 < x, y < 1 \\ 0, elsewhere \end{cases}$ Examine X and Y are independent.

4. Define joint distributions of two random variables X and Y and state its properties.

5. If the probability density function of X is $f_x(x)$=2x, 0<x<1, find the probability density function of Y=3X+1.

6. The joint probability density function of two random variables given by
$f_{xy}(x.y)$ = x(x-y)/8, 0<x<2; -x<y<x and find $f_{y/x}(y/x)$
7. If X and Y are random variables having the joint density function f(x,y) = (6-x-y)/8, 0<x<2; 2<y<4,find P(X+Y<3).
8. Can Y=5+2.8 X and X=3-0.5 Y be the estimated regression equations of Y on X and X on Y respectively? Explain your answer.
9. Show that $Cov^2(X,Y) \le Var(X).Var(Y)$
10. Prove that Cov(aX,bY)=ab.Cov(X,Y).
11. The two equations of the variables X and Y are x=19.13 - 0.87y and y=11.64 - 0.50x. Find the correlation co-efficient between X and Y.
12. Prove that the correlation coefficient Pxy takes value in the range -1 to 1.
13. Distinguish between correlation and regression.
14. Find the acute angle between the two lines of regression.
15.The two regression lines are x+6y=4, 2x+3y =1.Find the mean values of x and y.
16. State the equations of the two regression lines.
17. Write the applications of central limit theorem.
18. State central limit theorem in Liapounoff's form
19. State central limit theorem.
20. State central limit theorem in Lindberg-Levy's form

PART-B(16 Marks)

1.a. Given is the joint distribution X and Y

		X		
		0	1	2
Y	0	0.02	0.08	0.10
	1	0.05	0.20	0.25
	2	0.03	0.12	0.15

Obtain (i) Marginal Distributions and
(ii) The Conditional Distribution of X given Y=0. **(8)**

b. The joint probability mass function of X and Y is given by

P(x,y)		0	1	2
	0	0.1	0.04	0.02
X	1	0.08	0.20	0.06
	2	0.06	0.14	0.30

Compute the marginal PMF of X and Y,P[x≤1, y≤1] and check if X and Y are independent. **(8)**

2.a. If the joint probability density function of a two dimensional random variable (X,Y) is given by $f(x,y) = x^2 + \frac{xy}{3}$, 0<x<1,0<y<2
= 0, elsewhere
Find (i) P(X>1/2)
(ii) P(Y>X) and
(iii) P(Y<1/2/ X<1/2).
(iv)P(Y<1)

(v) Find the conditional density functions **(8)**

b. The joint probability mass function of (X,Y) is given by p(x,y) = K(2x+3y), x = 0,1,2; y =1,2,3. Find all the marginal and conditional probability distributions. **(8)**

3.a. The joint probability density function of the two dimensional random variable is

$f(x,y) = \frac{8}{9}xy$, $1<x<y<2$

= 0 , otherwise **(8)**

(i)Find the marginal density functions of X and Y

(ii) Find the conditional density function of Y given X

b. The joint pdf of a bivariate random variable (X,Y) is given by

$f(x,y)= kxy, 0<x<1, 0<y<1$

= 0 , otherwise

Find (i) k (ii) P(X+Y<1) (iii) Are X and Y independent **(8)**

4. a. Two random variable X and Y have the joint density

$f(x,y) = 2-x-y$; $0<x<1$, $0<y<1$

= 0, otherwise.

Show that Cov(X,Y) = -1/11. **(8)**

b. Suppose the joint probability density function is given by

$f(x,y) = \frac{6}{5}(x+y^2)$ $0\le x\le 1$, $0\le y\le 1$

= 0 , otherwise.

Obtain the marginal PDF of X and that of Y.

Hence or otherwise find $P[1/4\le y\le 3/4]$ **(8)**

5. a. If X and Y are random variables having the joint p.d.f.

$f(x,y)=\frac{1}{8}(6-x-y), 0<x<2, 2<y<4$. Find P[X<1/Y<3]. **(8)**

b. The joint density function of the random variables Xand Y is given by

$f(x,y)=8xy, 0<x<1, 0<y<x$

=0, elsewhere.

Find the marginal and conditional density functions. **(8)**

6.a. Given $f_{xy}(x,y)$ = cx(x-y), $0<x<2$, $-x<y<x$

= 0, otherwise

(1) Evaluate C

(2) Find $f_x(x)$

(3) $f_{y/x}(y/x)$ and

(4) $f_y(y)$ **(8)**

b. Find the coefficient of correlation between X and Y from the following data

X : 25	28	35	32	31	36	29	38	34	32
Y : 43	46	49	42	36	32	31	30	33	38

(8)

7.a. If the equations of the two lines of regression of y on x and x on y are respectively 7x-16y+9=0; 5y-4x-3=0, calculate the coefficient of correlation. **(8)**

b. Two random variables X and Y have the joint pdf

$f(x,y) = k(4-x-y)$, $0\le x\le 2$, $0\le y\le 2$

= 0 , otherwise

Find the correlation coefficient between X and Y. **(8)**

8.a. Obtain the two regression equations to the following data:

X: 65	66	67	67	68	69	70	72
Y: 67	68	65	68	72	72	69	71

(8)

b. If the joint p.d.f. of (X,Y) is given by f(x,y)=x+y,0≤x,y≤1,find the p.d.f. of U=XY. **(8)**

9.a. Let (X,Y) be a two-dimensional non-negative continuous random variable having the joint density f(x,y) = $\begin{cases} 4xye^{-(x^2+y^2)}, x \geq 0, y \geq 0 \\ 0, elsewhere \end{cases}$

Find the density function of U=$\sqrt{(X^2+Y^2)}$ **(8)**

b. The joint p.d.f of X and Y is given by f(x,y) = $e^{-(x+y)}$ x>0,y>0,find the probability density function of U = (X+Y)/2. **(8)**

10.a. State and prove central limit theorem.

b. A distribution with unknown mean µ has variance equal to 1.5. Use central limit theorem to find how large a sample should be taken from the distribution in order that the probability will be atleast 0.95 that the sample mean will be within 0.5 of the population mean. **(8)**

11.a. If $X_1,X_2,X_3,\ldots\ldots X_n$ are Poisson variates with mean 2,use central limit theorem to evaluate $P(120<s_n<160)$ where $S_n=X_1+X_2+X_3.\ \ldots.X_n$ and n=75. **(8)**

b. A sample of size 100 is taken from a population of mean 60 and variance 400. Using CLT find the probability that the sample mean will not differ from the population mean by more than four. **(8)**

UNIT – III
MARKOV PROCESSES AND MARKOV CHAIN
PART – A (2 Marks)

1. What do you understand by stationary process?
2. {X(s,t)} is a random process , what is the nature of X(s,t) when (a) s is fixed (b) t is fixed?
3. Define wide – sense stationary process.
4. Define strict sense stationary random process.
5. Consider the random process X(t) = cos (t+ϕ),where ϕ is uniformly distributed in $(-\pi/2,\pi/2)$. Check whether the process is stationary?
6. Give an example of Markov process.
7. Define a Markov process and a Markov chain.
8. State Chapman-Kolmogorov theorem.
9. Define irreducible Markov chain.
10. Prove that the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$ is the tpm of an irreducible Markov chain.
11. Define one-step Transition probability.
12. Define Transition probability matrix.
13. If the transition probability matrix of a Markov chain is $\begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$, find the limiting

distribution of the chain.

14. What is a stochastic matrix?when is it said to be regular?
15. The initial process of the Markovian transition probability matrix is given by

$$P = \begin{pmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.2 & 0.7 \\ 0.6 & 0.3 & 0.1 \end{pmatrix}$$ with initial probabilities $P_1(0) = 0.4$; $P_2(0) = 0.3$; $P_3(0) = 0.3$

find $P_1(1)$.

16. Show that Poisson process is not a stationary.
17. The additive property holds good for any number of independent Poisson process. Justify.
18. A radioactive source emits particles at a rate of 5 per minute in accordance with Poisson process,Each particles emitted has a probability 0.6 of being recorded. Find the probability that 10 particles are recorded in 4-minperiod.
19. If patients arrive at a clinic according to Poisson process with mean rate of 2 per minute. Find the probability that during a 1- minute interval no patient arrives.
20. The probability that a person is suffering from cancer is 0.001.Find the probability that out of 4000 persons.Exactly 4 suffer because of cancer.

PART – B(16 Marks)

1.a. Explain the classification of random process.Give an example to each case. **(8)**

b. Consider the random process $X(t) = \cos(\omega_0 t+\theta)$,where θ is uniformly distributed in the interval $-\pi$ to π.check whether X(t) is stationry or not. **(8)**

2.a. Show that the random process $X(t) = A\cos(\omega t+\theta)$ is wide sense stationary if A & ω are constant and θ is uniformly distributed random variable in (0.2π). **(8)**

b. Show that the process $X(t) = A\cos\lambda t+B\sin\lambda t$ where A & B are random variables is WSS,if (i) E(A)=E(B)=0,(ii) $E(A^2)=E(B^2)$ and (iii) E(AB) =0. **(8)**

3.a. Show that the random process $X(t) = A\sin(\omega t+\theta)$ where A and ω are constants and θ is a uniformly distributed in $(0,2\pi)$ is wide sense stationary. **(8)**

b. The process{X(t)} whose probability distribution under certain conditions is given by

$$P\{X(t)=n\} = \frac{(at)^{n-1}}{(1+at)^{n+1}}, n = 1,2,........$$

$$= \frac{at}{1+at}, n = 0.$$ Show that it is not stationary. **(8)**

4.a. The transition probability matrix of the Markov chain $\{X_n\}$, n = 1,2,3,……having 3 states 1,2 and 3 is $P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$ and the initial distribution is

$P^{(0)}$ =(0.7 0.2 0.1).Find (i)$P(X_2=3)$ and (ii) $P(X_3=2,X_2=3,X_1=3,X_0=2)$. **(8)**

b. A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again he is to travel by train. Now suppose that on the first day of the week the man tossed a fair die and drove to work if and only if a “6” appeared. Find (1) the probability that he takes a train on the third day. (2) the probability that he drives to work in the long run. **(8)**

5.a. The tpm of a Markov chain with 3 states 0,1,2 is P $\begin{pmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{pmatrix}$ and the initial state distribution of the chain is $P(X_0=i) = \frac{1}{3}$, i = 0,1,2. Find (i) $P(X_2=2)$ (ii) $P(X_3=1;X_2=2;X_1=1;X_0=2)$ and (iii) $P(X_1=1/X_2=2)$. **(8)**

b. Consider a Markov chain with state space {0,1} and the tpm P= $\begin{pmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

(i) Draw a transition diagram.
(ii) Show that state 1 is transient.
(iii) Is the chain irreducible. **(8)**

6.a. Find the nature of the states of the Markov chain with the transition probability matrix $\begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix}$. **(8)**

b. Suppose that the probability of a dry day following a rainy day is $\frac{1}{3}$ and that the probability of a rainy day following a dry day is $\frac{1}{2}$.Given that may 1 is a dry day. Find the probability that may 3 is a dry day and also may 5 is a dry day. **(8)**

7.a. Let $\{X_n : n=1,2,3,..\}$ be a Markov chain on the space S = {1,2,3} with one step transition probabilities P = $\begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 1 & 0 & 0 \end{pmatrix}$

(i) Sketch the transition diagram.
(ii) Is the chain irreducible?Explain.
(iii) Is the chain Ergodic?Explain. **(8)**

b. Three boys A,B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B as to A. Show that the process is Markovian and prove that the chain is irreducible. Find also the steady-state distribution of the chain. **(8)**

8.a. Prove that the sum of two independent Poisson processes is a Poisson process but the difference is not a Poisson process. **(8)**

b. If customers arrive at a counter in accordance with a poisson process with a mean rate of 2 per minute,find the probability that the interval between 2 consecutive arrivals is (i) more than 1 min (ii) between min.1 and min.2 and (iii) 4 min.or less. **(8)**

9.a. Suppose that customers arrive at a bank according to a Poisson process with a mean rate of 3 per minute ; find the probability that during a time interval of 2 min (i) exactly 4 customers arrive and (ii) more than 4 customers arrive. **(8)**

b. State the postulates of Poisson process and also prove that the inter-arrival time of Poisson process follows exponential distribution. **(8)**

10.a. Define the Poisson process and obtain its probability distribution. **(8)**

b. If $\{N_1(t)\}$ and $\{N_2(t)\}$ are two independent Poisson process with parameters λ_1 and λ_2 respectively, show that

$$P[N_1(t) = k / \{ N_1(t) + N_2(t) = n\}] = \binom{n}{k} p^k q^{n-k},$$

Where $p = \dfrac{\lambda_1}{\lambda_1 + \lambda_2}$ and $q = \dfrac{\lambda_2}{\lambda_1 + \lambda_2}$. **(8)**

UNIT-IV
QUEUEING THEORY
PART-A (2 Marks)

1. Define birth and death process.
2. What are the basic characteristics of a queueing system?
3. Define Kendall's notation.
4. Give the formulas for the waiting time of a customer in the queue and in the system for the (M/M/1):(∞/FIFO) model.
5. In the usual notation of an M/M/1 queueing system, if λ=3/hour and μ=4/hour, find $P(X \geq 5)$ where X is the number of customers in the system.
6. In the usual notation of an M/M/1 queueing system, if λ=12/hour and μ=24/hour, find the average number of customers in the system.
7. Derive the average number of customers in the system for (M/M/1):(∞/FIFO) model.
8. State Little's formula for an (M/M/1):(∞/FIFO) queueing model.
9. In a given (M/M/1):(∞/FCFS) queue, ρ=0.6, what is the probability that the queue contains 5 or more customers?
10. What is the probability that a customer has to wait more than 15 minutes to get his service completed in a (M/M/1):(∞/FIFO) queue system, if λ=6 per hour and μ=10 per hour?
11. Consider an M/M/1 queueing system. If λ=6 and μ=8, find the probability of atleast 10 customers in the system.
12. Consider an M/M/1 queueing system. Find the probability of finding atleast 'n' customers in the system.
13. What is the probability that an arrival to an infinite capacity 3 server Poisson queue with $\lambda/c\mu$ = 2/3 and P_0=1/9 enters the service without waiting?
14. Consider an M/M/C queueing system. Find the probability that an arriving customer is forced to join the queue.
15. For (M/M/C):(N/FIFO)model, write down the formula for
    (i) Average number of customers in the queue
    (ii)Average waiting time in the system
16. Give the formulas for the average number of customers in the queue and in the system for the (M/M/s):(∞/FIFO) queueing model.
17. What is the effective arrival rate for (M/M/1):(4/FCFS) queueing model when λ=2 and μ=5.

18. Give the probability that there is no customer in an (M/M/1):(k/FIFO) queueing system.
19. Write the formulas for the average number of customers in the (M/M/1):(k/FIFO) queueing system and also in the queue.
20. Define effective arrival rate with respect to an (M/M/1):(k/FIFO) and (M/M/s):(k/FIFO) queueing models.

PART-B(16 Marks)

1.a. Derive the balance equation of birth and death process. **(8)**

b. Discuss the pure birth process and hence obtain its probabilities, mean and variance.**(8)**

2.a. Arrivals at a telephone booth are considered to be poisson with an average time of 12 min. between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 min.
(a) Find the average number of persons waiting in the system.
(b) What is the probability that it will take him more than 10min. altogether to wait for the phone and complete his call?
(c) Estimate the fraction of the day when the phone will be in use
(d) What is the average length of the queue that forms time to time? **(8)**

b. In a railway marshalling yard goods-trains arrive at an average rate of 30 per day according to Poisson distribution. If the mechanic services according exponential distribution with a mean of 36 minutes, find L_s,L_q and W_s. **(8)**

3.a. Customers arrive at a one-man barber shop according to a Poisson process with a mean interarrival time of 12 min. Customers spend an average of 10min in the barber's chair.
(a) What is the expected number of customers in the barber shop and in the queue?
(b) How much time can a customer expect to spend in the barber's shop?
(c) What is the average time customer spends in the queue?
(d) What is the probability that more than 3 customers are in the system? **(8)**

b. A T.V. repairman finds that the time spent on his jobs follows exponential distribution with a mean of 30 minutes. If he repairs the sets in the order of their arrival according to Poisson distribution at an average rate of 10 per 8 hour-day, find his expected idle time on each day and also the total number of sets in his shop.**(8)**

4.a. Customers arrive at a watch repair shop according to a Poisson process at a rate of one per every 10 minutes, and the service time is an exponential random variable with mean 8 minutes. Find the average number of customers L_S, the average waiting time a customer spends in the shop W_S and the average time a customer spends in the waiting for service W_q. **(8)**

b. A duplicating machine maintained for office use is operated by an office assistant who earns Rs. 5 per hour. The time to complete each jobs varies according to an exponential distribution with mean 6 mins. Assume a Poisson input with an average arrival rate of 5 jobs per hour. If an 8-hrs day is used as a base , determine
(1) The percentage idle time of the machine.
(2) The average time a job is in the system.
(3) The average earning per day of the assistant. **(8)**

5.a. There are three typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour.
(a) what fraction of the time all the typists will be busy?
(b) What is the average number of letters waiting to be typed?
(c) What is the average time a letter has to spend for waiting and for being typed?

(d) What is the probability that a letter will take longer than 20 min. waiting to be typed and being typed? **(8)**

b. A petrol pump station has 4 pumps. The service times follow the exponential distribution with a mean of 6 min and cars arrive for service in a poisson process at the rate of 30 cars per hour.

(a) what is the probability that an arrival would have to wait in line?

(b) find the average waiting time, average time spend in the system and the average number of cars in the system.

(c) For what percentage of time would a pump be idle on an average? **(8)**

6.a. A supermarket has 2 girls attending to sales at the counters. If the service times for each customer is exponential with mean 4min and if people arrive in Poisson fashion at the rate of 10per hour,

(a) what is the probability that a customer has to wait for service?

(b) What is the expected percentage of idle time for each girl?

(c) If the customer has to wait in the queue, what is the expected length of his waiting time? **(8)**

b. A bank has two tellers working on savings account. Service time for each customer is 3 minutes and customers arrive at an average rate of 30 per hour. Assuming Poisson arrivals and exponential services, find the probability for a customer has to wait for service and also the expected waiting time. **(8)**

7.a. A tax consulting firm has three counters in its office to receive people who have problems concerning their income and sales taxes. On the average 48 persons arrive in an 8-hour day. Each tax advisor spends 15 minutes on the average on an arrival. If the arrivals are Poisson distributed and service times are exponentially distributed, find:

(a) Average number of customers in the system

(b) Average number of customers waiting to be served.

(c) Average time a customer spends in the system

(d) Average waiting time for a customer. **(8)**

b. The local one person barber shop can accommodate a maximum of 5 people at a time(4waiting and 1 getting hair-cut) Customers arrive according to a Poisson distribution with mean 5 per hour. The barber cuts hair at an average rate of 4 per hour.

(a) what percentage of time is the barber idle?

(b) What fraction of the potential customers are turned away?

(c) What is the expected number of customers waiting for a hair-cut?

(d) How much time can a customer expect to spend in the barber shop? **(8)**

8.a. Patients arrive at clinic according to poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour.

(a) find the effective arrival rate at the clinic.

(b) What is the probability that an arriving patient will not wait?

(c) What is the expected waiting time until a patients is discharged from the clinic? **(8)**

b. Customers arrive at a one window drive-in bank according to Poisson distribution with mean 10 per hour. Service time per customer is exponential with mean 5 minutes. The space in front of the window including that for the serviced car can accommodate a maximum of 3 cars. Other can wait outside this space.

(1) What is the probability that an arriving customer can drive directly to the space in front of the window?
(2) What is the probability that an arriving customer will have to wait outside the indicated space?
(3) How long the arriving customer is expected to wait before starting service? **(8)**

9.a. A 2-person barber shop has 5 chairs to accommodate waiting customers. Potential customers, who arrive when all 5 chairs are full, leave without entering barber shop. Customers arrive at the average rate of 4 per hour and spend an average of 12 min in the barber's chair. Compute P_0,P_1,P_7,$E[N_q]$ and $E[W]$. **(8)**

b. A car servicing station has 2 bays where service can be offered simultaneously. Because of space limitation, only 4 cars are accepted for servicing. The arrival pattern is poisson with 12 cars per day. The service time in both the bays is exponentially distributed with μ=8 cars per day per boy. Find the average number of cars in the service station, the average number of cars waiting for service and the average time a car spends in the system. **(8)**

10.a. For an M/M/2 queueing system with a waiting room of capacity 5, find the average number of customers in the system, assuming that arrival rate as 4 per hour and mean service time 30 minutes. **(8)**

b. In a machine repair station, the machine mechanic repairs four machines. Meantime between service requirements is 5 hours for each machine with Exponential distribution and mean repair time is one hour with exponential distribution. Find
(a) Probability that the service will be idle.
(b) Average number of machine waiting to be repaired and being repaired.
(c) Expected time a machine will wait in queue to be repaired. **(8)**

UNIT-V

NON-MARKOVIAN QUEUES AND QUEUE NETWORKS

PART-A (2 Marks)

1. Write Pollaczek-Khintchine formula and explain the notations.
2. What you mean by M/G/1 queue?
3. In an M/G/1/FCFS with infinite capacity queue, the arrival rate λ = 5 and the mean service time E(S)=1/8 hour and Var(S) = 0. Compute the mean waiting time W_q in the queue.
4. Define series queues.
5. Define series queues with blocking.
6. Define Jackson networks.
7. Define open Jackson networks.
8. Write the traffic equations in open Jackson networks.
9. Define closed Jackson networks.
10. Write the flow balance equations in closed Jackson networks.

PART-B(16 Marks)

1.a. Derive Pollaczek-Khinchine formula for the average number of customers in the M/G/1 queueing system. **(10)**

b. A one-man barber shop takes exactly 25 minutes to complete one hair-cut. If customers arrive at the barber shop in a Poisson fashion at an average rate of one every 40 minutes, how long on the average a customer spends in the shop? Also find the average time a customer must wait for service. **(6)**

2.a. Automatic car wash facility operates with only one boy. Cars arrive according to a Poisson process, with mean of 4 cars per hour and may wait in the facility's parking lot if the boy is busy. If the service time for all cars is constant and equal to 10 min, determine

(1) Mean number of customers in the system.
(2) Mean number of customers in the queue.
(3) Mean waiting time in the system.
(4) Mean waiting time in the queue. **(8)**

b. A car wash facility operates with only one boy. Cars arrive according to a Poisson distribution with mean a of 4 cars per hour and may wait in the facility's parking lot if the boy is busy. The parking lot is large enough to accommodate any number of cars. If the time for washing and cleaning a car follows normal distribution with mean 12 minutes and S.D 3 minutes, find the average number of cars waiting in the parking lot. Also find the mean waiting time of cars in the parking lot. **(8)**

3.a. A patient who goes a single doctor clinic for a general check-up has to go through 4 phases.The doctor takes on the average 4 minutes for each for each phase of the check-up and the time taken for each phase is exponentially distributed.If the arrivals of the patients at the clinic are approximately Poisson at the average rate of 3 per hour,what is the average time spent by a patient (i) in the examination? (ii) waiting in the clinic? **(8)**

b. In a car manufacturing plant, a loading crane takes exactly 10 minutes to load a car into wagon and again come back to position to load another car. If the arrival of cars is a Poisson stream at an average of 1 every 20 minutes, calculate the following

(i) The average number of cars in the system
(ii) The average number of cars in the queue
(iii) The average waiting time of a car in the system
(iv) The average waiting time of a car in the queue. **(8)**

4. A car wash facility operates with only one boy. Cars arrive according to a Poisson distribution with mean a of 4 cars per hour and may wait in the facility's parking lot if the boy is busy. The parking lot is large enough to accommodate any number of cars. Find the average number of cars waiting in the parking lot, if the time for washing and cleaning a car follows

(a) Uniform distribution between 8 and 12 minutes.
(b) a normal distribution with mean 12 minutes and S.D 3 minutes.
(c) a discrete distribution with values equal to 4, 8 and 15 minutes and corresponding probabilities 0.2, 0.6 and 0.2. **(16)**

5.a. Prove that in a series queues inter-departure times follows an exponential distribution with parameter λ. **(8)**

b. Derive the steady state probabilities on series queues with blocking. **(8)**

6. The supermarket owner is experimenting with a new store design and has remodeled one of his stores as follows. Instead of the usual checkout-counter design, the store has been remodeled to include a check out "lounge". As customers complete their shopping, they enter the lounge with their carts. If all checkers are busy the customers receive a number. They then park their carts and take a seat. When a checker is free, the next number is called and the customer having that particular number enters the available check out counters. The store has been enlarged so that for practical

purposes, there is no limit on either the number of shoppers that can be in the shopping section or the number that can wait in the lounge, even during peak periods. It has been estimated that during peak hours, customers arrive according to a Poisson process at a mean rate of 40/hr and it takes a customer on the average, $\frac{3}{4}$ hr to fill his shopping cart. The filling times are exponentially distributed. Furthermore, the checkout times are also approximately exponentially distributed with a mean of 4 minutes, irrespective of the particular checkout counter(during peak periods each counter has a cashier and bagger, hence the low mean checkout time). The management wishes to know the following:

1. Minimum number of checkout counters required in operation during peak periods.
2. If it is decided to add one more counter than the minimum number of counters required, then what is the average waiting time in the lounge?
3. How many people, on the average will be in the lounge?
4. How many people, on the average will be in the entire supermarket? **(16)**

7. a. There are 2 salesman in a supermarket. Out of the 2 salesman, one is incharge of billing and receiving payment while the other salesman is incharge of weighing and delivering the items. Due to lack of space, only if the billing clerk is free. The customer who has finished his billing job has to wait until the delivery section becomes free. If customers arrive according to a Poisson process at rate 1 per hr and the service times of 2 clerks are independent and have exponential rates of 3 per hr and 2 per hr. Find
   (i) The proportion of customers who enter the supermarket.
   (ii) The average number of customers in the supermarket.
   (iii) The average amount of time a customer spends in the shop. **(8)**

   b. There are 2 salesman in a supermarket. Out of the 2 salesman, one is incharge of billing and receiving payment while the other salesman is incharge of weighing and delivering the items. Due to lack of space, only if the billing clerk is free. The customer who has finished his billing job has to wait until the delivery section becomes free. If customers arrive according to a Poisson process at rate of 5 per hr and both the salesman take 6 per hr. Find
   (i) The proportion of customers who enter the supermarket.
   (ii) The average number of customers in the supermarket.
   (iii) The average amount of time a customer spends in the shop. **(8)**

8.a. A TVS company in MADURAI containing a repair section shared by a large number of machines has 2 sequential stations with respective service rates of 3 per hour and 4 per hour. The cumulative failure rate of all the machines is 1 per hour. Assuming that the system behaviour can be approximated by a 2-stage tandem queue, find:
   (i) The probability that both the service stations are idle
   (ii) The average repair time including the waiting time and
   (iii) The bottleneck of the repair facility. **(8)**

   b. Define open Jackson network and discuss for the steady state solution. **(8)**

9.a. In a book shop, there are 2 sections, one for engineering books and the other section for mathematics books. There is only one salesman in each section. Customers from outside arrive at the engineering book section at a Poisson rate of 4 per hour and at the mathematics book section at a Poisson rate of 3 per hour.

The service rates of the engineering book section and mathematics book section are 8 and 10 per hour respectively. A customers after service at engineering book section is equally likely to go to the mathematics book section or to leave the book shop. However, a customer upon completion of service at mathematics book section will go the engineering book section with probability $\frac{1}{3}$ and will leave the book shop otherwise. Find the

(1). Joint steady-state probability that there are 3 customers in the engineering book section and 2 in the mathematics book section.
(2). Average number of customers in the book shop.
(3). Average waiting time of a customer in the book shop. **(8)**

b. In a library, there are 2 sections, one for english books and the other section for tamil books. There is only one salesman in each section. Customers from outside arrive at the english book section at a Poisson rate of 5 per hour and at the tamil book section at a Poisson rate of 4 per hour. The service rates of the english book section and tamil book section are 9 and 11 per hour respectively. A customers after service at english book section is equally likely to go to the tamil book section or to leave the library. However, a customer upon completion of service at tamil book section will go the english book section with probability $\frac{1}{3}$ and will leave the library otherwise. Find the

(1). Joint steady-state probability that there are 2 customers in the english book section and 2 in the tamil book section.
(2). Average number of customers in the library.
(3). Average waiting time of a customer in the library. **(8)**

10. In Lucky silks, at Thanjavur, there are 3 sections,the first station containing saree section, second containing gents wear and the third on containing kids wear. Customers arrive at the saree section at the rate of 5 per hour, gents wear section at the rate of 10 per hour and the kids wear section at the rate of 15 per hour. The service times at the saree section is 10 per hour, gents wear section is 50 per hour, kids wear section is 100 per hour. There is only one salesman in each section.Saree section is equally likely to go to the gents wear section or kids wear section or leave the system. A customer departing from the gents wear section always go to the kids wear section. A customer who service is over at the kids wear section either go to the gents wear section or leave the shop, the probability being the same. Find

(1) Average number of customers in the shop.
(2) Average waiting time of a customer in the shop. **(16)**