KINGS

COLLEGE OF ENGINEERING
Punalkulam

DEPARTMENT OF MATHEMATICS

ACADEMIC YEAR 2010-2011 / EVEN SEMESTER

QUESTION BANK

SUBJECT NAME: NUMERICAL METHODS **YEAR/SEM: III/VI**

UNIT - I

SOLUTION OF EQUATIONS AND EIGENVALUE PROBLEMS

PART-A(2 Marks)

1. If f(x) = 0 has root between x = a and x = b, then write the first approximate root by the method of false position.
2. Find an approximate value of the root of $x^3-3x+1 = 0$ lying between 1 and 2 by regula falsi method.
3. Write Newton's formula to find the cube root of N.
4. State fixed point theorem (or) If g(x) is continous in [a,b], then under what condition in [a,b]?.
5. Write down the condition for the convergence of Gauss – Seidel iteration scheme.
6. State any two differences between direct and iterative methods for solving system of equations.
7. Write the formula for finding $\sqrt{N}$ where N is a real number, by Newton's method.
8. State the criterion for the convergence in Newton Raphson method.
9. Distinguish Gauss Elimination method and Gauss Jordan method.
10. Solve the following system of equations by Gauss – Elimination method.
    5x + 4y = 15, 3x + 7 y = 12.
11. When Gauss-Elimination method fails?
12. What is the condition for the convergence of Gauss – Seidal method?
13. When will the solution of AX = B by Gauss – Seidel method converge quickly?
14. Compare Gauss elimination and Guass - Seidal Methods.
15. What are the two types of errors involving in the numerical computations?
16. Define Round off error.
17. Define Truncation error.
18. Using Gauss-Jordan method, solve the following system of equations
    5x +4y = 15 , 3x +7y = 12.
19. Find the dominant eigen value of A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ by power method?
20. How will you find the smallest eigen value of a square matrix A?

PART – B(16 Marks)

1. a. Find the real root of the equation $X \log_{10} X - 1.2 = 0$ correct to four places of decimal using false position method **(8)**
   b. Solve the equation $3x + \sin x - e^x = 0$ by Regula falsi method. **(8)**

2. a. Find the positive root of $x^3 = 2x + 5$ by the method of False Position. **(8)**
   b. Find the iteration formula to find $\sqrt{N}$ where N is a positive integer by Newton's method and hence find $\sqrt{11}$
3. a. Find to 4 decimals by Newton's Method, a root of $x^{\sin 2} - 4$. **(8)**
   b. Derive a Newton-Raphson iteration formula for finding the cube root of a positive number N. Hence find $\sqrt[3]{12}$ **(8)**
4. a. Obtain an iteration formula, using N - R values to find the reciprocal of a given number N and hence find $\frac{1}{19}$, correction of 4 decimal places. **(8)**
   b. Find the double root of $x^3 - x^2 - x - 1 = 0$ choosing with the initial value of 0.8. **(8)**
5. a. Solve the system of equations 10x-2y+3z=23; 2x+10y-5z=-33; 3x-4y+10z=41 using Gauss – elimination method. **(8)**
   b. Solve the following system of equation using Gauss – elimination method 2x + y + 4z = 12 , 8x -3y +2z = 20 , 4x +11y – z =33. **(8)**
6. a. Using Gauss-Jordan method, solve the following system of equations 2x –y +2z = 8 , -x +2y + z = 4, 3x + y -4z = 0 . **(8)**
   b. Using Gauss-Jordan method, solve the following system of equations 3x+4y-7z =23, 7x-y+2z = -14, x+10y-2z = 33. **(8)**
7. a. Solve the following system of equations by Gauss-Jacobi Method 27x+6y-z=85; x+y+54z=110; 6x+15y+2z=72.
   b. Using Gauss – Seidel method, solve the following system. Start with x = 1, y = -2, z = 3. x+3y+52z = 173.61; x-27y+2z = 71.31; 41x-2y+3z = 65.46 **(8)**
8. a. Solve the system of equations 10x-5y-2z=3; x+6y+10z=-3; 4x-10y+3z=-3 by Gauss – Seidel Method. **(8)**
   b. Using Gauss-Jordon method, find the inverse of the matrix $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$ **(8)**
9. a. Using Gauss-Jordan Method find the inverse of the Matrix $\begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \\ 1 & 2 & 2 \end{pmatrix}$. **(8)**
   b. Determine the largest eigen value and the corresponding eigen vector correct to 3 decimal places, using power method for the matrix A = $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ **(8)**

10.a. Find the numerically largest eigen values of A = $\begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$ by power method corresponding eigen vector (correct to 3 decimal places). Start with initial eigen value $\begin{pmatrix}1\\0\\0\end{pmatrix}$. **(8)**

b. Find the largest eigen value and eigen vector of the matrix by power Method $\begin{bmatrix}1 & 2 & 3\\0 & -4 & 2\\0 & 0 & 7\end{bmatrix}$. **(8)**

UNIT II
INTERPOLATION AND APPROXIMATION

PART- A (2 Marks)

1. Explain the use of Lagrange's interpolation formula.
2. Obtain the Lagrange's interpolating polynomial for the observed data of points (1,1),(2,1) and (3,-2).
3. Write the Lagrange's formula to find y if three sets of values $(x_{0,} y_0)$, (x_1, y_1) and $(x_{2,} y_2)$ are given.
4. Given f(0) = -1, f(1) = 1 and f(2) = 4. Find the Newton's interpolating formula.
5. State any two properties of divided differences.
6. What are the n^{th} divided differences of a polynomial of the n^{th} degree.
7. State Newton's divided difference interpolation formula.
8. Find the divided differences of f(x) = x^2+x+2 for the arguments 1,3,6,11.
9. If f(x) = (1/x^2) find f(a,b) and f(a,b,c) by using divided differences.
10. Show that the divided differences are symmetrical in their arguments.
11. Construct the divided difference table

x :	3	6	8
y :	-4	5	10

12. What is meant by natural cubic spline.
13. State the conditions required for a natural cubic spline.
14. When will we use Newton's forward interpolation formula.
15. When will we use Newton's backward interpolation formula.
16. State Newton – Gregory forward difference interpolation formula.
17. Find the Polynomial for the following data by Newton's backward difference formula:

X:	0	1	2	3
F(x)	-3	2	9	18

18. Find the value of Y at x =21 using Newton's forward difference formula from the following table:

X:	20	23	26	29
Y:	0.3420	0.3907	0.4384	0.4848

19. Find f(2.5) from the data:

X:	1	2	3
F(x)	0	1	8

20. Find the sixth term in the sequence 8,12,19,29,42,......

PART – B(16 Marks)

1. a. Find the Lagrange's polynomial of degree 3 to fit the data:
 y(0)=-12, y(1)=0,y(3)=6 and y(4) =12. Hence find y(2). **(8)**

 b. Using Lagrange's interpolation formula find f(4) given that f(0) = 2, f(1) = 3, f(2) = 12 , f(15) = 3587. **(8)**

2. a. Using Lagrange's formula, fit a polynomial to the data **(8)**

x	0	1	2	4	5	6
F(x)	1	14	15	5	6	19

 Also find f(3).

 b. Using Lagrange's formula, fit a polynomial to the data **(8)**

X:	-1	0	2	3
Y:	-8	3	1	12

 Hence find y at x=1.5 and x=1.

3. a. Find the missing term in the following table using Lagrange's interpolation **(8)**

X	0	1	2	3	4
y	1	3	9	-	81

 b. Using Lagrange's formula, fit a polynomial to the data **(8)**

X:	2	5	7	10	121
Y:	18	180	448	1210	2028

 Hence find y at x=6.

4. a. If f(0) = 0 , f(1) = 0, f(2) = -12, f(4) =(0), f(5) =600 and f(7) = 7308,find a polynomial that satisfies this data using Newton's divided difference interpolation formula. Hence find f(6). **(8)**

 b. Find f(x) and f(6) from the following data: **(8)**

X:	0	2	3	4	7	9
f(x)	4	26	58	112	466	922

5. a. Using Newton's divided difference formula find the value of f(8) and f(6) from the following data: **(8)**

X:	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

 Also find f(12).

 b. Using Newton's divided difference formula find the cubic function of x. from the following data: **(8)**

X:	0	1	4	5
f(x)	8	11	68	123

6. Using cubic spline, find y(0.5) and y(1.5) from the following data, assuming that y''(0)=0 and y''(2)=0 **(16)**

x	0	1	2
y	-5	-4	3

7. Find the cubic spline for the data:

x	1	2	3	4
f(x)	1	2	5	11

 Assume that y''(1)=0 and y''(4)=0. **(16)**

8. Obtain the cubic spline approximation for the function y=f(x) from the following data, given that y''(-1)=0 and y''(2)=0. **(16)**

x	-1	0	1	2
y	-1	1	3	35

9. a. From the following table of half-yearly premium for policies maturing at different ages, estimate the premium for policies maturing at age 46 and 63. **(8)**

AgeX:	45	50	55	60	65
PremiumY:	114.84	96.16	83.32	74.48	68.48

b. From the following table, find the value of tan 45◦15' by Newton's forward interpolation formula **(8)**

X◦:	45	46	47	48	49	50
tanx◦	1.00000	1.03553	1.07237	1.11061	1.15037	1.19175

10. a. Find the polynomial for the data:

X:	1	2	3	4	5	6	7	8
f(x):	1	8	27	64	125	216	343	512

Estimate f(7.5). Use Newton's formula. **(8)**

b. The following data are taken from the steam table:

Temp.C	140	150	160	170	180
Pressure Kgf/cm^2	3.685	4.854	6.302	8.076	10.225

Find the pressure at temperature t = 142^0 and t = 175^0. **(8)**

UNIT – III
NUMERICAL DIFFERENTIATION AND INTEGRATION

PART – A(2 Marks)

1. In numerical integration, what should be the number of intervals to apply Simpson's 1/3 rule and by Simpson's 3/8 rule?
2. Compare Trapezoidal rule and Simpson's 1/3 rule for evaluating numerical integration.
3. State Simpson's 1/3 rule formula to evaluate $\int_a^b f(x)dx.$
4. Using Trapezoidal rule evaluate $\int_0^\pi \sin x dx$ by dividing the range into 6 equal parts.
5. State Romberg's method integration formula to find the value of I = $\int_a^b f(x)dx.$, using h and h/2.
6. Write down the Simpson's 3/8 rule of integration given (n+1) data.
7. Given f(0)=-1,f(1)=1 and f(2)=4, find $\int_0^2 f(x)dx$ by Trapezoidal rule.
8. If I_1 = 0.775, I_2 = 0.7828 find I using Romberg's method.
9. Using Simpson's rule find $\int_0^4 e^x dx.$ given $e^0 = 1$, $e^1 = 2.72$, $e^2 = 7.39$, $e^3 = 20.09$ and

$e^4 = 54.6$

10. What is the local error term in Trapezoidal formula and in Simpson's one third rule?
11. What is the order of the error in trapezoidal rule?
12. Evaluate $\int_0^1 \frac{dx}{1+x}$ with h = 0.5 using Trapezoidal rule.
13. A curve passes through (2,8), (3,27), (4,64) and (5,125) Find the area of the curve between the x axis and the lines x = 2 and x = 5 by Trapezoidal rule.
14. Find $\int_{-2}^{2} x^4 dx.$ by Simpson's rule taking h = 1.
15. If I = $\int_0^1 e^{-x2} dx.$ then $I_1 = 0.731$, $I_2 = 0.7430$ with h = 0.5 and h = 0.25 Find I using Romberg's method.
16. From the following table find the area bounded by the curve and the x axis from x = 2 to x = 7

x	2	3	4	5	6	7
f(x)	8	27	64	125	216	343

17. State two point Gaussian Quadrature formula to evaluate $\int_{-1}^{1} f(x)d(x).$
18. Evaluate $I = \int_0^1 \frac{dt}{1+t^2}$ by using three point Gaussian quadrature formula.
19. State Trapezoidal rule for evaluating $\int_a^b \int_c^d f(x,y)dxdy.$
20. State Simpson's rule for evaluating $\int_a^b \int_c^d f(x,y)dxdy.$

PART - B(16 Marks)

1. a. Find the value of f'(8) from the following table **(8)**

x :	6	7	9	12
$f(x)$:	1.556	1.690	1.908	2.158

b. Find the first and second derivative of the function $f(x) = x^3 - 9x - 14$ at x = 3.0 using the value given below:

x:	3.0	3.2	3.4	3.6	3.8	4.0
$f(x)$:	-14	-10.03	-5.296	-0.256	-6.672	14

(8)

2. a. Find f '(4) and f ''(4) from the table

x :	0	2	3	5
f(x) :	8	6	20	108

(8)

b. Given that

x :	1.1	1.2	1.3	1.4	1.5
y :	8.403	8.781	9.129	9.451	9.75

(8)

find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 1.1

3. a. The table given below reveals the velocity v of a body during the time 't' specified. Find its acceleration at t =1.1

t :	1.0	1.1	1.2	1.3	1.4
v :	43.1	47.7	52.1	56.4	60.8

(8)

b. By dividing the range into equal parts, evaluate $\int_0^{\pi} \sin x dx$ by using Trapezoidal rule. **(8)**

4. a. A river is 80 mts wide. The depth 'd' in mts at a distance x mts from one bank is given by the following table. Calculate the area of cross section of the river using Simpson's $\frac{1}{3}$rd rule

x :	0	10	20	30	40	50	60	70	80
d :	0	4	7	9	12	15	14	8	3

(8)

b. Use Simpson's $\frac{1}{3}$ rule to estimate the value of $\int_1^5 f(x)dx$ given

x :	1	2	3	4	5
$f(x)$:	13	50	70	80	100

(8)

5. a. Find the value of $\log_e 2$ from $\int_0^1 \frac{x^2 dx}{1+x^3}$ using Simpson's one third rule with R = 0.25. **(8)**

b. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by Simpson's $\frac{3}{8}$ rule. **(8)**

6. a. Evaluate $\int_1^{1.4} e^{-x^2} dx$ by taking h =0.1 using Simpson's $\frac{3}{8}$ rule. **(8)**

b. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Romberg's method. Hence obtain an approximate value of π. **(8)**

7. a. Evaluate $\int_0^2 \frac{dx}{x^2+4}$ using Romberg's method. Hence obtain an approximate value of π.

(8)

b. Evaluate I = $\int_{-1}^{1} \frac{dx}{1+x^2}$ by Three point Gaussian Quadrature formula. **(8)**

8. a. Using Gaussian Three point formula, compute $\int_5^{12} \frac{dx}{x}$, correct to four decimal places. **(8)**

b. Evaluate $I = \int_0^1 \frac{dx}{1+x}$ using two point Gaussian Quadrature formula. **(8)**

9. a. Evaluate $\int_1^2\int_1^2 \frac{dxdy}{x+y}$ by using Simpson's $\frac{1}{3}$rd rule taking Δx = Δy = 0.25 **(8)**

b. Evaluate $\int_0^2\int_0^1 4xydxdy$ by using Simpson's rule and taking h=$\frac{1}{4}$ and k =$\frac{1}{2}$. **(8)**

10. a. Evaluate $\int_1^{1.2}\int_1^{1.4} \frac{dxdy}{x+y}$ by Trapezoidal rule taking h=0.1 and k=0.1. **(8)**

b. Use Trapezoidal rule to evaluate I = $\int_1^2\int_1^2 \frac{dxdy}{x+y}$ taking 4 subintervals. **(8)**

UNIT – IV
INITIAL VALUE PROBLEMS FOR ORDINARY DIFFERENTIAL EQUATIONS

PART – A(2 Marks)

1. In solving dy/dx = f(x,y) , $y(x_0) = y_0$, write down Taylor's series for $y(x_1)$.
2. Write the merits and demerits of the Taylor method?
3. Which is better Taylor's method or R.K. method?
4. Solve the differential equation $\frac{dy}{dx} = x + y + xy, y(0) = 1$ by Taylor series method to get the value of y at x = h?
5. What is the truncation error in Taylor's series?
6. Given dy/dx = x+y , y(0) =1 find y(0.1) by Taylor series.
7. Write down Euler algorithm (formula) to the differential equation $\frac{dy}{dx} = f(x, y)$.
8. State modified Euler algorithm to solve y ' = f(x,y), $y(x_0)=y_0$ at $x = x_0 + h$.
9. Using Modified Euler's method, find y(0.1) if $\frac{dy}{dx} = x^2 + y^2, y(0) = 1$
10. What is the Error in modified Euler's method?
11. Find the values of k_1 and l_1 to solve y"+xy'+y=0,y(0)=1,y'(0)=0 by fourth order R.K method.
12. Write the Runge-Kutta algorithm of second order for solving y' = f(x,y), $y(x_0)=y_0$
13. State the third order R.K. method to find the numerical solution of the first order differential equation?
14. Write down the Runge-Kutta formula of fourth order to solve dy/dx = f(x,y) with $y(x_0) = y_0$
15. What are the advantages of R.K. method over Taylor method?
    (or)
    Why is R.K. method preferred to Taylor series method?
    (or)
    Compare Taylor's series and R.K. method?
16. Write Milne's predictor corrector formula?

17. What is the error of Milne's method?
18. How many prior values are required to predict the next value in Milne's method?
19. Write down Adams-Bashforth predictor formula?
20. What is a Predictor-Collector method of solving a differential equation?

PART – B(16 Marks)

1. a. Using Taylor series method find y at x = 0.1 if $\frac{dy}{dx} = x^2 y - 1, y(0) = 1.$ **(8)**

   b. Solve $\frac{dy}{dx} = x^2 + y^2, y(0) = 1.$ Using Taylor series at x=0.2 and 0.4. **(8)**

2. a. Using Taylor series method find y at x = 0.1,0.2 correct to four decimal places from $\frac{dy}{dx} = x^2 - y, y(0) = 1.$ with h = 0.1. **(8)**

   b. Solve $y' = x + y, y(1) = 0,$ by Taylor's series method. Find y(1.1). **(8)**

3. a. Find the Taylor series solution with three terms for the initial value problem $y' = x^3 + y, y(1) = 1.$ Find y at x=1.1 **(8)**

   b. Using Taylor series method with the first five terms in the expansion find y(0.1) correct to three decimal places, given that $y' = e^x - y^2, y(0) = 1.$ **(8)**

4. a. Using Taylor series method, find y(1.1) and y(1.2) correct to four decimal places given. $\frac{dy}{dx} = xy^{1/3}$ and y(1) =1. **(8)**

   b. Using Euler's method find y(0.2) and y(0.4) from $y' = x + y, y(0) = 1,$ with h=0.2. **(8)**

5. a. Using Euler's modified method find y(0.1) from $y' = x + y + xy, y(0) = 1,$ with h=0.05. **(8)**

   b. Using Euler's method find y(0.3) if y(x) satisfies the initial value problem. $y' = \frac{1}{2}(x^2 + 1)y^2, y(0.2) = 1.1114,$ with h=0.2 **(8)**

6. a. Using modified Euler's method, compute y(0.1) with h=0.1 from $\frac{dy}{dx} = y - \frac{2x}{y}, y(0) = 1.$

   b. Solve $y' = 2xy, y(0) = 1,$ compute y at x=0.25 by modified Euler's method. **(8)**

7. a. Given $\frac{dy}{dx} = x^3 + y, y(0) = 2.$ Compute y(0.2), y(0.4) and y(0.6) by Runge-Kutta method of fourth order. **(8)**

   b. Using R.K. method of 4th order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}, y(0) = 1.$ at x = 0.2. **(8)**

8. a. Using R.K. method of fourth order find y(0.2) for the initial value problem $\frac{dy}{dx} = \log(x + y), y(0) = 1.$ taking h=0.1 **(8)**

   b. Find y(0.8) given that $y' = y - x^2, y(0.6) = 1.7379,$ by using R.K.method of 4th order. **(8)**

9. a. Consider the second order initial value problem $y'' - 2y' + 2y = e^{2t} \sin t$ with y(0) = -0.4 and $y'(0) = -0.6$ using fourth order R.K.method, find y(0.2). **(8)**

   b. Apply the fourth order Runge-Kutta method , to find an approximate value of y when x = 0.2 and x= 0.4, given that $y' = x + y, y(0) = 1,$ with h=0.2. **(8)**

10. a. Given $\frac{dy}{dx} = x^3 + y, y(0) = 2.$ The values of y(0.2) = 2.073,y(0.4)=2.452 and y(0.6) = 3.023 are got by R.K. method of fourth order. Find y(0.8) by Milne's predictor-corrector method taking h=0.2. **(8)**

b. Using Milne's method find y(0.4) given $\frac{dy}{dx} = 1 - y, y(0) = 0;.$,y(0.1) = 0.1.Obtain y(0.2) by improved euler method and y(0.3) by R.K method of fourth order. **(8)**

11. Determine the value of y(0.4) using Milne's method given $\frac{dy}{dx} = y^2 + xy, y(0) = 1;.$ use Taylor series to get the values of y(0.1), y(0.2) and y(0.3). **(16)**

12. Using Runge-Kutta method of order 4, find y for x=0.1,0.2,0.3 given that $\frac{dy}{dx} = y^2 + xy, y(0) = 1;.$ and also find the solution at x=0.4 using Milne's method. **(16)**

13.a. Given $\frac{dy}{dx} = x^2(1 + y), y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979,$ evaluate y(1.4) by Adams – Bashforth method**.** **(8)**

b. Given $\frac{dy}{dx} = y - x^2, y(0) = 1;.$ y(0.2) = 1.1218 y(0.4) = 1.4682 y(0.6) = 1.7379 estimate y(0.8) by Adam's method **(8)**

14. Consider the initial value problem $\frac{dy}{dx} = y - x^2 + 1, y(0) = 0.5.$ **(16)**

a. Using the modified Euler method, find y(0.2)
b. Using 4th order Runge-Kutta method, find y(0.4) and y(0.6)
c. Using Adam-Bashforth Predictor- Corrector method. Find y(0.8).

UNIT V

BOUNDARY VALUE PROBLEMS IN ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS

PART – A(2 Marks)

1. Write the finite difference scheme of the differential equation $y''+y=0$.
2. State the conditions for the equation. $Au_{xx} + Bu_{yy} + Cu_{yy} + Du_x + Eu_y + Fu = G$ where A, B, C, D, E, F, G are function of x and y to be (i) elliptic (ii) parabolic (iii) hyperbolic
3. State the condition for the equation $Au_{xx}+2Bu_{xy}+Cu_{yy}=f(u_x,u_y,x,y)$ to be (a)elliptic (b) parabolic (c) hyperbolic when A, B, C are functions of x and y
4. What is the classification of $f_x-f_{yy}=0$?
5. Give an example of a parabolic equation?
6. State Schmidt's explicit formula for solving heat flow equation?
7. Write an explicit formula to solve numerically the heat equation(parabolic equation) $u_{xx}-au_t=0$?
8. What is the value of k to solve $\frac{\partial u}{\partial t} = \frac{1}{2} u_{xx}$ by Bender-Schmidt method with h=1 if h and k are the increments of x and t respectively?
9. What is the classification of one dimensional heat flow equation?

10. Write down the Crank-Nicholson formula to solve $u_t=u_{xx}$?
(or)
Write down the implicit formula to solve one dimensional heat flow equation?
11. Write the Crank Nicholson difference scheme to solve $u_{xx}=au_t$ with $u(0,t) = T_0$, $u(l,t)=T_1$ and the initial condition as $u(x,0) = f(x)$?
12. Write a note on the stability and convergence of the solution of the difference equation corresponding to the hyperbolic equation $u_{tt}=a^2u_{xx}$
13. Write down the general and simplest forms of the difference equation corresponding to the hyperbolic equation $u_{tt}=a^2u_{xx}$.
14. Write the difference scheme for solving the Laplace equation?
point difference formula for $\nabla^2\phi= 0$ is
15. Write the diagonal five-point formula to solve the Laplace equation $u_{xx}+u_{yy}=0$?
16. Write down the standard five point formula to solve Laplace equation
17. What is the purpose of Liebmann's process?
18. If u satisfies Laplace equation and u = 100 on the boundary of a square what will be the value of u at an interior grid point?
19. State Liebmann's iteration process formula?]
20. State the general form of Poisson's equation in partial derivatives?

PART – B(16 Marks)

1. a. Solve by finite difference method, the boundary value problem
$y''(x) - y(x) = 2$ where $y(0) = 0$ and $y(1) = 1$, taking $h = ¼$. **(8)**

b. Solve $\frac{d^2y}{dx^2} = xy$ given y(0)= -1 , y(1) = 2 by finite difference method with h = $\frac{1}{2}$ **(8)**

2. a. Using finite difference method, solve $\frac{d^2y}{dx^2} = y$ in (0,2) given y(0) = 0 , y(2) = 3.63 subdividing the range of x into 4 equal parts. . **(8)**

b. Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0\le x\le 1$, $t\ge 0$ with u(x,0)=x(1-x), 0<x<1 and u(0,t)=u(1,t) = 0, for all t >0 using explicit method with $\Delta x=0.2$ for 3 time steps. **(8)**

3. a. Solve $\frac{\partial^2 u}{\partial x^2} = 2\frac{\partial u}{\partial t}$ given u(0,t)=0, u(4,t)=0,u(x,0)=x(4-x) taking $\Delta x=\Delta t=1$. Find the value of u upto t=3 using Bender-Schmidt explicit difference scheme **(8)**

b. Using Schmidt's process solve 25 $u_{xx}=u_t$ where 0<x<1 t>0 with boundary conditions u(0,t)=0=u(10,t); u(x,0)= $\frac{x(10-x)}{25}$ and choosing h=1 and k suitably. Find $u_{i,j}$ for i=1,2,3….9 and j= 1,2,3,4 **(8)**

4. a. By Cranck-Nicholson method solve the equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ subject to u(x,0)=0, u(0,t) = 0 and u(1,t)=t for two time steps. **(8)**

b. Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ in 0<x<5, $t \ge 0$ given that u(x,0) =20, u(0,t)=0,u(5,t) =100.
Compute u for the time-step with h=1 by Cranck – Nicholson method **(8)**

5.a. Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$,0<x<2,t>0,u(0,t)=u(2,t)=0,t>0 and u(x,0)= $\sin\frac{\pi x}{2}$,$0 \le x \le 2$, and Δt=0.25 and Δx=0.5 for two times steps by Crank-Nicholson implicit finite difference method **(8)**

b. Solve $\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}$, 0<x<1.t>0 given u(x,0)=0, $\frac{\partial}{\partial t}(x,0)$ =u(0,t)=0 and u(1,t)=100sinπt, complete u(x,t) for 4 times steps with h=0.25 **(8)**

6. a. Solve $y_{tt} = y_{xx}$ upto t 0.5 with a spacing of 0.1 subject to y(0,t) = 0 , y(1,t) = 0 , $y_t(x,0) = 0$ and y(x,0) = 10+ x(1-x) **(8)**

b. Solve numerically , $4u_{xx}= u_{tt}$ with the boundary conditions u(0,t) =0 , u(4,t) = 0 and the initial conditions $u_t(x,0) = 0$ and u(x,0) = x(4-x),taking h=1.(for 4 time steps) **(8)**

7. Solve the Laplace's equation over the square mesh of side 4 units satisfying the boundary conditions: **(16)**

U(0,y)=0,$0 \le y \le 4$;u(4,y)=12+y, $0 \le y \le 4$

U(x,0)=3x, $0 \le x \le 4$;u(x,4)= x^2 ,$0 \le x \le 4$

8. By iteration method , solve the laplace equation $u_{xx}+u_{yy}=0$, over the square region , satisfying the boundary condition

U(0,y)=0,$0 \le y \le 3$;u(3,y)=9+y, $0 \le y \le 3$

U(x,0)=3x, $0 \le x \le 3$;u(x,3)= 4x,$0 \le x \le 3$ **(16)**

9. Solve $\nabla^2 u = -10(x^2 + y^2 + 10.)$ over the square mesh with sides x=0,y=0,x=3,y=3 with u=0 on the boundary and mesh length of 1 unit. **(16)**

10. Solve the Poisson equation $u_{xx} + u_{yy} = -x^2 y^2$ over the square region bounded by the lines x=0,y=3 given that u=10 throughout the boundaries taking h=1. **(16)**

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