KINGS

COLLEGE OF ENGINEERING

DEPARTMENT OF MATHEMATICS
ACADEMIC YEAR 2010-2011 / EVEN SEMESTER

QUESTION BANK

SUBJECT NAME: NUMERICAL METHODS **YEAR/SEM: II / IV**

UNIT - I
SOLUTION OF EQUATIONS AND EIGENVALUE PROBLEMS

PART-A(2 Marks)

1. What is the condition for the convergence of the iteration method for solving $x = \varphi(x)$?
2. What is the order of convergence for fixed point iteration?
3. State the fixed point iteration theorem.
4. Write Newton's formula to find the cube root of N.
5. Write the formula for finding $\sqrt{N}$ where N is a real number, by Newton's method.
6. State the criterion for the convergence in Newton Raphson method
7. What is the rate of convergence in N.R method?
8. Distinguish Gauss Elimination method and Gauss Jordan method.
9. Solve the following system of equations by Gauss – Elimination method.
10. Write down the condition for the convergence of Gauss – Seidel iteration scheme.
11. Distinguish between direct and iterative (indirect) method of solving simultaneous equations.
12. When will the solution of AX = B by Gauss – Seidel method converge Quickly?
13. Give an example of (a) algebraic (b) transcendental equation.
14. What are the two types of errors involving in the numerical computations?
15. Define Round off error.
16. Define Truncation error
17. Using Gauss-Jordan method, solve the following system of equations $5x + 4y = 15$, $3x + 7y = 12$
18. Find the dominant eigenvalue of $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ by power method?
19. Using Jacobi method, find eigen values of $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$
20. How will you find the smallest eigen value of a square matrix A?

PART – B(16 Marks)

1. a. Find the real root of the equation $3x - \cos x - 2 = 0$ by iteration method, correct to three places of decimals. **(8)**

   b. Find the cube root of 15, correct to four decimal places, by iteration method. **(8)**

2. a. Find the iteration formula to find $\sqrt{N}$ where N is a positive integer by Newton's method and hence find $\sqrt{11}$ **(8)**

b. Find to 4 decimals by Newton's Method, a root $x^{\sin 2} - 4$. **(8)**

3. a. Show that Newton-Raphson formula to find $\sqrt{a}$ can be in the form

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right), \text{ a = 0,1,2,…}$$ **(8)**

b. Derive a Newton-Raphson iteration formula for finding the cube root of a positive number N. Hence find $\sqrt[3]{12}$ **(8)**

4. a. Obtain an iteration formula, using N - R values to find the reciprocal of a given number N and hence find $\frac{1}{19}$, correction of 4 decimal places **(8)**

b. Find the negative root of the equation $x^2 + 4\sin x = 0$ by Newton-Raphson method correct to three decimal places **(8)**

5. a. Find the double root of $x^3 - x^2 - x - 1 = 0$ choosing with the initial value of 0.8. **(8)**

b. Solve the following system of equation using Gauss – elimination method
2x + y + 4z = 12 , 8x -3y +2z = 20 , 4x +11y – z =33. **(8)**

6. a. Using Gauss-Jordan method, solve the following system of equations
2x –y +2z = 8 , -x +2y + z = 4, 3x + y -4z = 0 . **(8)**

b. Using Gauss-Jordan method, solve the following system of equations
3x+4y-7z =23, 7x-y+2z = -14, x+10y-2z = 33. **(8)**

7. a. Using Gauss – Seidel method, solve the following system. Start with x = 1, y = -2, z = 3 , x+3y+52z = 173.61; x-27y+2z = 71.31; 41x-2y+3z = 65.46 **(8)**

b. Solve the following system by Gauss – Seidel method
10x-5y-2z = -3; 4x-10y+3z = -3; x+6y+10z = 3 **(8)**

8. a. Using Gauss-Jordon method , find the inverse of the matrix

$$\begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$$ **(8)**

b. Find the inverse of the given matrix by Gauss Jordon method A = $\begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ **(8)**

9. a. Find the numerically largest eigen values of A = $\begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$ by power method corresponding eigen vector (correct to 3 decimal places). Start with initial eigen value $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ **(8)**

b. Determine the largest eigen value and the corresponding eigen vector correct to 3 decimal places, using power method for the matrix A = $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ **(8)**

10.a. Find the eigen values and eigen vectors of the matrix A = $\begin{pmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{pmatrix}$

by using Jacobi method. **(8)**

b. Find the eigen values and eigen vectors of the matrix A = $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$ by using

Jacobi method. **(8)**

UNIT II
INTERPOLATION AND APPROXIMATION

PART- A (2 Marks)

1. Explain the use of Lagrange's interpolation formula.
2. Obtain the Lagrange's interpolating polynomial for the observed data of points (1,1),(2,1) and (3,-2).
3. Find the polynomial which takes the following values

X:	0	1	2
Y:	1	2	1

4. Given f(0) = -1, f(1) = 1 and f(2) = 4. Find the Newton's interpolating formula.
5. State any two properties of divided differences.
6. What are the n^{th} divided differences of a polynomial of the n^{th} degree.
7. State Newton's divided difference interpolation formula.
8. Find the divided differences of $f(x) = x^2+x+2$ for the arguments 1,3,6,11.
9. If $f(x) = (1/x^2)$ find f(a,b) and f(a,b,c) by using divided differences.
10. Show that the divided differences are symmetrical in their arguments.
11. What is a cubic spline.
12. What is a natural cubic spline.
13. State the conditions required for a natural cubic spline.
14. When will we use Newton's forward interpolation formula
15. When will we use Newton's backward interpolation formula.
16. State Newton – Gregory forward difference interpolation formula.
17. Find the polynomial for the following data by Newton's backward difference formula:

X:	0	1	2	3
F(x)	-3	2	9	18

18. Find the value of Y at x =21 using Newton's forward difference formula from the following table:

X:	20	23	26	29
Y:	0.3420	0.3907	0.4384	0.4848

20. Find the sixth term in the sequence 8,12,19,29,42,......

PART – B(16 Marks)

1. a. Find the Lagrange's polynomial of degree 3 to fit the data: **(8)**
   Y(0)=-12, y(1)=0,y(3)=6 and y(4) =12. Hence find y(2).
   b. Using Lagrange's interpolation formula find f(4) given that f(0) = 2, f(1) = 3, f(2) = 12, f(15) = 3587. **(8)**
2. a. Using Lagrange's formula, fit a polynomial to the data **(8)**

X:	0	1	2	4	5	6
F(x):	1	14	15	5	6	19

Also find f(3).

   b. Using Lagrange's formula, fit a polynomial to the data **(8)**

X:	-1	0	2	3
Y:	-8	3	1	12

Hence find y at x=1.5 and x=1

3. a. Using Lagrange's formula, fit a polynomial to the data **(8)**

X:	0	1	3	4
Y:	-12	0	6	12

Hence find y at x=2.

   b. Using Lagrange's formula, fit a polynomial to the data **(8)**

X:	2	5	7	10	121
Y:	18	180	448	1210	2028

Hence find y at x=6.

4. a. If f(0) = 0 , f(1) = 0, f(2) = -12, f(4) =(0), f(5) =600 and f(7) = 7308,find a polynomial that satisfies this data using Newton's divided difference interpolation formula., Hence, find f(6). **(8)**
   b. Using Newton's divided difference formula find f(x) and f(6) from the following data: **(8)**

X:	1	2	7	8
f(x)	1	5	5	4

5. a. Using Newton's divided difference formula find the value of f(8) and f(6) from the following data: **(8)**

X:	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

Also find f(-2) and f(12).

   b. Using Newton's divided difference formula find the cubic function of x from the following data: **(8)**

X:	0	1	4	5
f(x)	8	11	68	123

6. a. Using Newton's divided difference formula find the cubic function of x from the following data: **(8)**

X:	0	1	4	5
f(x)	2	3	12	147

b. Given the following table ,find f(2.5) using cubic spline functions: **(8)**

I:	0	1	2	3
Xi:	2	3	12	147
f(Xi):	0.5	0.3333	0.25	0.2

7. a. The following values of X and Y are given: **(8)**

X:	1	2	3	4
Y:	1	2	5	11

Find the cubic splines and evaluate Y(1.5).

b. Find the cubic splines for the data : **(8)**

X:	0	1	2	3
f(x):	1	2	9	28

8. a. Find the cubic splines for the data : **(8)**

X:	1	2	3
f(x):	-6	-1	16

b. Find the polynomial of degree two for the data by Newton's forward difference method: **(8)**

X:	0	1	2	3	4	5	6	7
F(x)	1	2	4	7	11	16	22	29

9. a. From the following table of half-yearly premium for policies maturing at different ages, estimate the premium for policies maturing at age 46 and 63. **(8)**

AgeX:	45	50	55	60	65
PremiumY:	114.84	96.16	83.32	74.48	68.48

b. From the following table, find the value of tan 45◦15' by Newton's forward interpolation formula **(8)**

X◦:	45	46	47	48	49	50
tanx◦	1.00000	1.03553	1.07237	1.11061	1.15037	1.19175

10. a. Given **(8)**

X:	1	2	3	4	5	6	7	8
f (x):	1	8	27	64	125	216	343s	512

Estimate f(7.5) . Use Newton's formula.

b. Construct Newton's forward interpolating polynomial for the following data: **(8)**

X:	4	6	8	10
Y:	1	3	8	16

UNIT – III
NUMERICAL DIFFERENTIATION AND INTEGRATION

PART – A(2 Marks)

1. In numerical integration, what should be the number of intervals to apply Simpson's 1/3 rule and by Simpson's 3/8 rule?
2. Compare Trapezoidal rule and Simpson's 1/3 rule for evaluating numerical integration

3. State Simpson's 1/3 rule formula to evaluate $\int_a^b f(x)dx.$

4. Using Trapezoidal rule evaluate $\int_0^\pi \sin x dx$ by dividing the range into 6 equal parts.

5. State Romberg's method integration formula to find the value of I = $\int_a^b f(x)dx.$, using h and h/2.

6. Write down the Simpson's 3/8 rule of integration given (n+1) data.

7. State the formula for Trapezoidal rule of integration.

8. If $I_1 = 0.775$, $I_2 = 0.7828$ find I using Romberg's method.

9. Using Simpson's rule find $\int_0^4 e^x dx.$ given $e^0 = 1$, $e^1 = 2.72$, $e^2 = 7.39$, $e^3 = 20.09$ and $e^4 = 54.6$

10. What is the local error term in Trapezoidal formula and in Simpson's one third rule?

11. What is the order of the error in trapezoidal rule ?

12. Evaluate $\int_0^1 \frac{dx}{1+x}$ with h = 0.5 using Trapezoidal rule

13. A curve passes thro' (2,8), (3,27), (4,64) and (5,125) Find the area of the curve between the x axis and the lines x = 2 and x = 5 by Trapezoidal rule.

14. Find $\int_{-2}^{2} x^4 dx.$ by Simpson's rule taking h = 1.

15. If I = $\int_0^1 e^{-x2} dx.$ then $I_1 = 0.731$, $I_2 = 0.7430$ with h = 0.5 and h = 0.25 Find I using Romberg's method.

16. From the following table find the area bounded by the curve and the x axis from x = 2 to x = 7

x	2	3	4	5	6	7
f(x)	8	27	64	125	216	343

17. State two point Gaussian Quadrature Formula to evaluate $\int_{-1}^{1} f(x)d(x).$

18. State three point Gaussian Quadrature Formula to evaluate $\int_{-1}^{1} f(x)d(x).$

19. State Trapezoidal rule for evaluating $\int_a^b \int_c^d f(x,y)dxdy.$

20. State Simpson's rule for evaluating $\int_a^b \int_c^d f(x,y)dxdy.$

PART - B(16 Marks)

1. a. Find the value of f '(8) from the following table **(8)**

x :	6	7	9	12
$f(x)$:	1.556	1.690	1.908	2.158

b. Find the first and second derivative of the function $f(x) = x^3 - 9x - 14$ at x = 3.0 using the value given below:

x:	3.0	3.2	3.4	3.6	3.8	4.0
$f(x)$:	-14	-10.03	-5.296	-0.256	-6.672	14

(8)

2. a. Using the given data find f '(5)

x :	0	2	3	4	7	9
f(x) :	4	26	58	112	466	922.

(8)

b. Given the values

x :	5	7	11	13	17
f(x) :	150	392	1452	2366	5202

Evaluate f(9) using Newton's divided difference formula **(8)**

3. a. Find f '(4) and f ''(4) from the table

x :	0	2	3	5
f(x) :	8	6	20	108

(8)

b. Given that

x :	1.1	1.2	1.3	1.4	1.5
y :	8.403	8.781	9.129	9.451	9.75

(8)

find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 1.1

4. a. Find f '(6) and the maximum value of y = f(x) given the data

x :	0	2	3	4	7	9
f(x) :	4	26	58	112	466	992.

(8)

b. Obtain the first and second derivatives of y at x = 0.96 from the data

x :	0.96	0.98	1.00	1.02	1.04
y :	0.7825	0.7739	0.7651	0.7563	0.7473

(8)

5. a. Find $\frac{dy}{dx}$ at x = 1.5 given

x :	1	2	3	4	5
y :	77	78	127	248	375

(8)

b. Find the first and second derivatives of y w. r. to x at x = 10

x :	3	5	7	9	11
y :	31	43	57	41	27

(8)

6. a. A river is 80 mts wide. The depth 'd' in mts at a distance x mts from one bank is given by the following table. Calculate the area of cross section of the river using Simpson's $\frac{1}{3}$rd rule

x : 0	10	20	30	40	50	60	70	80
d : 0	4	7	9	12	15	14	8	3

(8)

b. The table given below reveals the velocity v of a body during the time 't' specified. Find its acceleration at t =1.1

t : 1.0	1.1	1.2	1.3	1.4
v : 43.1	47.7	52.1	56.4	60.8

(8)

7. a. Use Simpson's $\frac{1}{3}$rd rule to estimate the value of $\int_1^5 f(x)dx$ given

x	:	1	2	3	4	5
$f(x)$	:	13	50	70	80	100

(8)

b. By dividing the range into 10 equal parts, evaluate $\int_0^{\pi} \sin x dx$ by using Simpson's $\frac{1}{3}$rd rule. **(8)**

8. a. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Romberg's method

Hence obtain an approximate value of π. **(8)**

b. Evaluate $\int_0^2 \frac{dx}{x^2+4}$ using Romberg's method

Hence obtain an approximate value of π. **(8)**

9. a. Evaluate I = $\int_{-1}^{1} \frac{dx}{1+x^2}$ by Three point Gaussian Quadrature formula. **(8)**

b. Evaluate $I = \int_0^1 \frac{dx}{1+x}$ using two point Gaussian Quadrature formula. **(8)**

10. a. Evaluate $\int_1^2\int_1^2 \frac{dxdy}{x+y}$ by using Simpson's $\frac{1}{3}$rd rule taking Δx = Δy = 0.25 **(8)**

b. Evaluate $\int_1^{1.2}\int_1^{1.4} \frac{dxdy}{x+y}$ by Trapezoidal rule taking h=0.1 and k=0.1. **(8)**

UNIT – IV
INITIAL VALUE PROBLEMS FOR ORDINARY DIFFERENTIAL EQUATIONS

PART – A(2 Marks)

1. State the disadvantage of Taylor series methods?
2. Write down the fourth order Taylor Algorithm?
3. Write the merits and demerits of the Taylor method of solution?
4. Which is better Taylor's method or R.K. method?
5. State Taylor series algorithm for the first order differential equation?
6. Solve the differential equation $\frac{dy}{dx} = x + y + xy,\ y(0)=1$ by Taylor series method to get the value of y at x = h?
7. What is the truncation error in Taylor's series?
8. Write down Euler algorithm to the differential equation $\frac{dy}{dx} = f(x,y)$.
9. State modified Euler algorithm to solve y ' = f(x,y), $y(x_0)=y_0$ at $x = x_0 + h$.
10. Using Modified Euler's method, ifnd y(0,1) if $\frac{dy}{dx} = x^2 + y^2,\ y(0)=1$

11. What is the error of Euler's method?
12. What are the limitations of Euler's method?
13. What is the Error in modified Euler's method?
14. Write the Runge-Kutta algorithm of second order for solving y' = f(x,y), $y(x_0)=y_0$
15. State the third order R.K. method algorithm to find the numerical solution of the first order differential equation?
16. Write down the Runge-Kutta formula of fourth order to solve dy/dx = f(x,y) with $y(x_0) = y_0$
17. State the special advantage of Runge-Kutta method over Taylor series method? What are the advantages of R.K. method over Taylor method?
    (or)
    Why is R.K. method preferred to Taylor series method?
    (or)
    Compare Taylor's series and R.K. method?
18. Write Milne's predictor corrector formula?
19. What is the error of Milne's method?
20. Write down Adams-Bashforth predictor formula?
21. What will you do, if there is a considerable difference between predicted value and corrected value, in predictor corrector methods?
22. Compare R.K. methods and Predictor-Corrector methods for solution of initial value problems
23. What is a Predictor-Collector method of solving a differential equation?
24. What is the Error of Adam Bashforth method?

PART – B(16 Marks)

1. a. Using Taylor series method find y at x = 0.1 if $\frac{dy}{dx} = x^2 y - 1, y(0) = 1.$ **(8)**

   b. Solve $\frac{dy}{dx} = x^2 + y^2, y(0) = 1.$ Use Taylor series at x=0.2 and 0.4, Find x = 0.1. **(8)**

2. a. Using Taylor series method find y at x = 0.1 correct to four decimal places from $\frac{dy}{dx} = x^2 - y, y(0) = 1.$ with h = 0.1. Compute terms upto x^4. **(8)**

   b. Solve $y' = x + y, y(1) = 0,$ by Taylor's series method. Find y(1.1). **(8)**

3. a. Find the Taylor series solution with three terms for the initial value problem $y' = x^3 + y, y(1) = 1.$ **(8)**

   b. Using Taylor series method with the first five terms in the expansion find y(0.1) correct to three decimal places, given that $y' = e^x - y^2, y(0) = 1.$ **(8)**

4. a. Using Taylor series method, find y(1.1) and y(1.2) correct to four decimal places given. $\frac{dy}{dx} = xy^{1/3}$ and y(1) =1. **(8)**

   b. Using Euler's method find y(0.2) and y(0.4) from $y' = x + y, y(0) = 1,$ with h=0.2. **(8)**

5. a. Using Euler's modified method find y(0.1) from $y' = x + y + xy, y(0) = 1,$ with h=0.05. **(8)**

   b. Using Euler's method find y(0.3)of y(x) satisfies the initial value problem $y' = \frac{1}{2}(x^2 + 1)y^2, y(0.2) = 1.1114,$ with h=0.2 **(8)**

6. a. Using modified Euler's method, compute y(0.1) with h=0.1 from $\frac{dy}{dx} = y - \frac{2x}{y}, y(0) = 1.$ **(8)**

b. Solve $y' = 1 - y, y(0) = 0,$ by modified Euler's method. **(8)**

7. a. Given $\frac{dy}{dx} = x^3 + y, y(0) = 2.$ Compute y(0.2), y(0.4) and y(0.6) by Runge-Kutta method of fourth order. **(8)**

b. Using R.K. method of 4th order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}, y(0) = 1.$ at x = 0.2. **(8)**

8. a. Using R.K. method of fourth order find y(0.1) for the initial value problem $\frac{dy}{dx} = \frac{xy}{1 + x^2}, y(0) = 1.$,take h=0.1 **(8)**

b. Find y(0.8) given that $y' = y - x^2, y(0.6) = 1.7379,$ by using R.K.method of 4th order. **(8)**

9. a. Consider the second order initial value problem $y'' - 2y' + 2y = e^{2t} \sin t$ with y(0) = -0.4 and $y'(0) = -0.6$ using fourth order R.K.method, find y(0.2). **(8)**

b. Apply the fourth order Runge-Kutta method , to find an approximate value of y when x = 0.2 and x= 0.4, given that $y' = x + y, y(0) = 1,$ with h=0.2. **(8)**

10. a. Given $\frac{dy}{dx} = x^3 + y, y(0) = 2.$ The values of y(0.2) = 2.073,y(0.4)=2.452 and y(0.6) = 3.023 are got by R.K. method of fourth order. Find y(0.8) by Milne's predictor-corrector method taking h=0.2. **(8)**

b. Using Milne's method find y(4.4) given $5xy' + y - 2 = 0$ given y(4)=1, y(4.1) = 1.0049,y(4.2)=1.0097 and y(4.3) = 1.0143. **(8)**

11. Determine the value of y(0.4) using Milne's method given $\frac{dy}{dx} = y^2 + xy, y(0) = 1;.$ use Taylor series to get the values of y(0.1), y(0.2) and y(0.3). **(16)**

12. Using Runge-Kutta method of order 4, find y for x=0.1,0.2,0.3 given that $\frac{dy}{dx} = y^2 + xy, y(0) = 1;.$ and also find the solution at x=0.4 using Milne's method. **(16)**

13. a. Given $\frac{dy}{dx} = x^2(1 + y), y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979,$ evaluate y(1.4) by Adams – Bashforth method. **(8)**

b. Using the above predictor-corrector equations, evaluate y(.4) , if y satisfies $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$ and y(1) = 1 , y(1.1) = 0.996, Y91.3) = 0.972 **(8)**

14. Consider the initial value problem $\frac{dy}{dx} = y - x^2 + 1, y(0) = 0.5.$ **(16)**

a. Using the modified Euler method, find y(0.2)
b. Using 4th order Runge-Kutta method, find y(0.4) and y(0.6)
c. Using Adam-Bashforth Predictor- Corrector method. Find y(0.8).

UNIT V
BOUNDARY VALUE PROBLEMS IN ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS

PART – A(2 Marks)

1. State the conditions for the equation.
   $Au_{xx} + Bu_{yy} + Cu_{yy} + Du_x + Eu_y + Fu = G$ where A, B, C, D, E, F, G are function of x and y to be (i) elliptic (ii) parabolic (iii) hyperbolic
2. State the condition for the equation $Au_{xx}+2Bu_{xy}+Cu_{yy}=f(u_x,u_y,x,y)$ to be (a)elliptic (b) parabolic (c) hyperbolic when A, B, C are functions of x and y
3. What is the classification of $f_x-f_{yy}=0$?
4. Give an example of a parabolic equation?
5. State Schmidt's explicit formula for sloving heat flow equation?
6. Write an explicit formula to solve numerically the heat equation(parabolic equation) $u_{xx}- au_t=0$?
7. What is the value of k to solve $\frac{\partial u}{\partial t} = \frac{1}{2} u_{xx}$ by Bender-Schmidt method with h=1 if h and k are the increments of x and t respectively?
8. What is the classification of one dimensional heat flow equation?
9. Write down the Crank-Nicholson formula to solve $u_t=u_{xx}$?
   (or)
   Write down the implicit formula to solve one dimensional heat flow equation?
10. What type of equations can be solved by using Crank-Nicholson's difference formula?
11. Write the Crank Nicholson difference scheme to solve $u_{xx}=au_t$ with $u(0,t) = T_0$, $u(l,t)=T_1$ and the initial condition as $u(x,0) = f(x)$?
12. For what purpose Bender-Schmidt recurrence relation is used?
13. Write a note on the stability and convergence of the solution of the difference equation corresponding to the hyperbolic equation $u_{tt}=a^2u_{xx}$
14. State the explicit scheme formula for the solution of the wave equation?
15. Write down the general and simplest forms of the difference equation corresponding to the hyperbolic equation $u_{tt}=a^2u_{xx}$.
16. For what value of λ, the explicit, method of solving the hyperbolic equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ is stable, where $\lambda = C\Delta t/\Delta x$?
17. Write the diagonal five-point formula to solve the Laplace equation $u_{xx}+u_{yy}=0$?
18. Write down the standard five point formula to solve Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$
19. Write the difference scheme for solving the Laplace equation?
    point difference formula for $\nabla^2\phi= 0$ is
20. What is the purpose of Liebmann's process?
21. If u satisfies Laplace equation and u = 100 on the boundary of a square what will be the value of u at an interior grid point?
22. Write the Laplace equations $u_{xx}+u_{yy} = 0$ in difference quotients?
23. Define a difference quotient?

24. State Liebmann's iteration process formula?
25. Write down the finite difference form of the equation $\nabla^2 u = f(x,y)$
26. Write the difference scheme for $\nabla^2 u = f(x,y)$?
27. State the five point formula to solve the poisson equation $u_{xx} + u_{yy} = 100$?
28. State the general form of Poisson's equation in partial derivatives?
29. What is Shooting method?
30. What is the procedure of shooting method?

PART – B(16 Marks)

1. a. Solve by finite difference method, the boundary value problem
$y''(x) - y(x) = 2$ where $y(0) = 0$ and $y(1) = 1$, taking $h = ¼$. **(8)**

b. Solve $\frac{d^2y}{dx^2} = xy$ given y(0)= -1 , y(1) = 2 by finite difference method with h = $\frac{1}{2}$ **(8)**

2. a. Using finite difference method, solve $\frac{d^2y}{dx^2} = y$ in (0,2) given y(0) = 0 , y(2) = 3.63 subdividing the range of x into 4 equal parts. . **(8)**

b. Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 \le x \le 1$, $t \ge 0$ with u(x,0)=x(1-x), 0<x<1 and u(0,t)=u(1,t) = 0, for all t >0 using explicit method with Δx=0.2 for 3 time steps. **(8)**

3. a. Solve $\frac{\partial^2 u}{\partial x^2} = 2\frac{\partial u}{\partial t}$ given u(0,t)=0, u(4,t)=0,u(x,0)=x(4-x) taking $\Delta x = \Delta t = 1$. Find the value of u upto t=3 using Bender-Schmidt explicit difference scheme **(8)**

b. Using Schmidt's process solve 25 $u_{xx} = u_t$ where 0<x<1 t>0 with boundary conditions u(0,t)=0=u(10,t); u(x,0)= $\frac{x(10-x)}{25}$ and choosing h=1 and k suitably. Find $u_{i,j}$ for i=1,2,3….9 and j= 1,2,3,4 **(8)**

4. a. By Cranck-Nicholson method solve the equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ subject to u(x,0)=0, u(0,t) = 0 and u(1,t)=t for two time steps**.** **(8)**

b. Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ in 0<x<5, $t \ge 0$ given that u(x,0) =20, u(0,t)=0,u(5,t) =100. Compute u for the time-step with h=1 by Cranck – Nicholson method **(8)**

5. a. Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$,0<x<2,t>0,u(0,t)=u(2,t)=0,t>0 and u(x,0)= $\sin\frac{\pi x}{2}$,$0 \le x \le 2$, and Δt=0.25 and Δx=0.5 for two times steps by Crank-Nicholson implicit finite difference method **(8)**

b. Solve $\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}$, 0<x<1.t>0 given u(x,0)=0, $\frac{\partial}{\partial t}(x,0)$ =u(0,t)=0 and u(1,t)=100sinπt, complete u(x,t) for 4 times steps with h=0.25 **(8)**

6. a. Solve $y_{tt} = y_{xx}$ upto t 0.5 with a spacing of 0.1 subject to y(0,t) = 0 , y(1,t) = 0 , $y_t(x,0) = 0$ and y(x,0) = 10+ x(1-x) **(8)**

b. Solve numerically , $4u_{xx} = u_{tt}$ with the boundary conditions u(0,t) =0 , u(4,t) = 0 and the initial conditions $u_t(x,0) = 0$ and u(x,0) = x(4-x),taking h=1.(for 4 time steps) **(8)**

7. Solve the Laplace's equation over the square mesh of side 4 units satisfying the boundary conditions**:** **(16)**

   U(0,y)=0,$0 \le y \le 4$;u(4,y)=12+y, $0 \le y \le 4$

   U(x,0)=3x, $0 \le x \le 4$;u(x,4)= x^2,$0 \le x \le 4$

8. By iteration method , solve the laplace equation $u_{xx}+u_{yy}=0$, over the square region , satisfying the boundary condition

   U(0,y)=0,$0 \le y \le 3$;u(3,y)=9+y, $0 \le y \le 3$

   U(x,0)=3x, $0 \le x \le 3$;u(x,3)= 4x,$0 \le x \le 3$ **(16)**

9. Solve $\nabla^2 u = -10(x^2 + y^2 + 10.)$ over the square mesh with sides x=0,y=0,x=3,y=3 with u=0 on the boundary and mesh length of 1 unit**.** **(16)**

10. Solve the Poisson equation $u_{xx} + u_{yy} = -x^2 y^2$ over the square region bounded by the linesx=0,y=3 given that u=10 throughout the boundaries taking h=1**.** **(16)**

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