KINGS
COLLEGE OF ENGINEERING
Punalkulam

DEPARTMENT OF MATHEMATICS
ACADEMIC YEAR 2010-2011 (Even Semester)

QUESTION BANK

SUBJECT CODE: MA1151
SUBJECT NAME: MATHEMATICS - II **YEAR / SEM: I / II**

UNIT- I
LAPLACE TRANSFORMS

PART-A

1. Write a function for which Laplace transformation doesnot exist. Explain why Laplace transform does not exist.
2. If L(f(t)) = F(s) what is $L(e^{-at}f(t))$?
3. Find the Laplace transform of $e^{-2t}(1+t)^2$.
4. Find the Laplace transform of $te^{-t}\sin t$.
5. Find L(tsin2t).
6. state the condition for the existence of Laplace Transfrom of f(t).
7. Find the Laplace transform of $\frac{1}{\sqrt{t}}$.
8. Obtain the Laplace transform of sin2t-2tcos2t in the simplified form.
9. Find the Laplace transform of $\frac{e^{-t}-e^{-3t}}{t}$.
10. If L(f(t)) = F(s), then show that L(f(at)) = $\frac{1}{a}F(\frac{s}{a})$.
11. Verify the initial value theorem for f(t) = 5 + 4cos2t.
12. State the final value theorem for Laplace transforms.
13. If L{f(t)}=$\frac{1}{s(s+2)}$, find $\lim_{t\to\infty}$ f(t).
14. Find the inverse Laplace transform of $\frac{1}{(s+2)^4}$.
15. State the convolution theorem for Laplace transforms.
16. Find the inverse Laplace Transfrom of $\frac{1}{s^2-4s+13}$.
17. Find the inverse Laplace transform of $\frac{s+2}{(s^2+4s+5)^2}$.
18. If L(f(t)) = $\frac{s+2}{s^2+4}$. Find the value of $\int_0^\infty f(t)dt$.

19. Find $L^{-1}(\frac{1}{2}\log_e(\frac{s+a}{s-a}))$.

20. Find the Laplace transform of t coshat.

PART-B

1. a) Find the Laplace transform of (i) $te^{-4t}\sin 3t$ and (ii) $\frac{e^{-t}\sin 2t}{t}$. **(8)**

 b) Find (i) $L\left(\frac{1-\cos t}{t}\right)$ and (ii) $L^{-1}\left[\cot^{-1}\left(\frac{s}{k}\right)\right]$. **(8)**

2. a) Find the Laplace transform of $e^{3t}(t\cos 2t)$ and $\frac{1-e^{-2t}}{t}$. **(8)**

 b) Find $L(t.e^{at}.\cos bt)$ and $L^{-1}[\tan^{-1}(s/a)]$. **(8)**

3. a) Verify the initial and final value theorems for the function $1+e^{-2t}$. **(8)**

 b) Find the inverse Laplace transform of $\frac{4(s-1)}{(s^2+2s+7)^2}$. **(8)**

4. a) Find the inverse Laplace transform of $\frac{1}{s(s^2+a^2)}$. **(8)**

 b) Find $L^{-1}\left[\frac{s+3}{(s+1)(s-2)(s+2)}\right]$ **(8)**

5. a)Using Convolution theorem to find $L^{-1}\left[\frac{s^2}{(s^2+1)(s^2+4)}\right]$ **(8)**

 b) Using convolution theorem find the inverse Laplace transform of $\frac{s+2}{(s^2+4s+13)^2}$. **(8)**

6. a) Using Laplace transform, solve: $\frac{d^2y}{dt^2}-3\frac{dy}{dt}+2y=4,\ y(0)=2,\ y'(0)=3.$ **(8)**

 b)Solve the differential equation, using Laplace transform y”+5y’+6y=2 given that y(0)=0 and y’(0)=0. **(8)**

7. a) Using Laplace transform solve y”+2y’-3y=3 given that y(0)=4 and y’(0)= -7. **(8)**

 b)Using Laplace transform solve $\frac{d^2y}{dt^2}+\frac{dy}{dt}=t^2+2t$ given that y=4 and y’= -2 when t=0. **(8)**

8. a) Using Laplace transform solve y”-2y’+y=e^t given that y(0)=2 and y’(0)=1. **(8)**

 b) Solve using Laplace transform $\frac{d^2y}{dt^2}-3\frac{dy}{dt}+2y=e^{3t}$ give that y(0)=1 . **(8)**

9. a) Find the Laplace transform of $f(t)=\begin{cases}\sin t, & 0<t<\pi \\ 0, & \pi<t<2\pi\end{cases}$ and f(t+2π) = f(t). **(8)**

 b) Find the Laplace transform of $f(t)=\begin{cases}E, & 0<t<a/2 \\ -E, & a/2<t<a\end{cases}$ and f(t+a) = f(t) **(8)**

10. a) Find the Laplace transform of $f(t) = \begin{cases} t, & 0<t<2 \\ 4-t, & 2<t<4 \end{cases}$, $f(t+4) = f(t)$. **(8)**

b) Find the Laplace transform of $f(t) = \begin{cases} a\sin\omega t, & 0<t<\pi/\omega \\ 0, & \pi/\omega<t<2\pi/\omega \end{cases}$ and $f(t+2\pi/\omega) = f(t)$. **(8)**

11. a) Find the Laplace transformof $f(t) = \begin{cases} k \text{ in}, & 0<t<a \\ -k \text{ in}, & a<t<2a \end{cases}$ and $f(t+2a) = f(t)$. **(8)**

b) Obtain the Laplace transform of $f(t) = \begin{cases} \cos t, & 0<t<2\pi \\ 0, & t>2\pi \end{cases}$ and $f(t+2\pi) = f(t)$. **(8)**

12. a) Using Laplace transform, solve: $\frac{dy}{dt} + 2y + \int_0^t y(t)dt = 0, y(0) = 1.$ **(8)**

b) Solve using Laplace transform, $y + \int_0^t ydt = 1 - e^t$. **(8)**

UNIT- II
VECTOR CALCULUS

PART – A

1. If $\vec{F} = (x^2 - y^2 + 2xz)\vec{i} + (xz - xy + yz)\vec{j} + (z^2 + x^2)\vec{k}$, find $grad\,(div\,\vec{F})$.
2. Prove that $\nabla(\log r) = \frac{\vec{r}}{r^2}$ where r=I $\vec{r}$ I
3. Find $\nabla r^n = nr^{n-2}.\vec{r}$ Where $r = \sqrt{x^2 + y^2 + z^2}$.
4. Find $\nabla.\left(\frac{1}{r}\vec{r}\right)$
5. Find the unit normal to the surface xyz=2 at (2,1,1).
6. If $f = x^2 + y^2 + z^2 - 8$, then find $grad f$ at $(2,0,2)$.
7. Define solenoidal vector and irrotational vector
8. If $\vec{V} = (x + 3y)\vec{i} + (y - 2z)\,\vec{j} + (x + l\,z)\vec{k}$ is solenoidal, find l.
9. For what value of k is the vector $r^k\,\vec{r}$ solenoidal.
10. Find a ,b,c so that the vector $\vec{F} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational.
11. Is the vector $x\vec{i} + 2y\vec{j} + 3z\vec{k}$, Irrotational?
12. Show that $\vec{F} = (x^2\vec{i} + y^2\vec{j} + z^2)\vec{k}$ is a conservative vector field.
13. If $\nabla\,\vec{F} = yz\,\vec{i} + xz\,\vec{j} + xy\,\vec{k}$ then find $\vec{F}$
14. If $\vec{F} = 5xy\vec{i} + 2y\vec{j}$, evaluate $\int_c \vec{F}.dr$ Where C is the part of the curve $y = x^2$ between $x = 1$ and $x = 2$.
15. Find $\iint_s \vec{r}.d\vec{s}$ Where S is the surface of the tetrahedron whose vertices are $(0,0,0), (1,0,0), (0,1,0), (0,0,1)$.

16. If S is any closed surface enclosing a volume V and $\vec{F}=ax\vec{i}+by\vec{j}+cz\vec{k}$, Prove that $\iint_S \vec{F}.\hat{n}ds$ =(a+b+c) V
17. State Green's theorem in a plane.
18. Using Green's theorem, Prove that the area enclosed by a simple closed curve C is $\frac{1}{2}\int(xdy-ydx)$
19. State Gauss divergence theorem.
20. State Stoke's theorem.

PART - B

1. a If $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$ and $r=\left|\vec{r}\right|$. Prove that $div\left(r^n\vec{r}\right)=(n+3)r^n$ and $curl\left(r^n\ \vec{r}\right)=0.$ **(8)**

   b If $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$, then prove that $div\,grad\left(r^n\right)=n(n+1)r^{n-2}$.Hence deduce that $divgrad\left(\frac{1}{r}\right)=0$ **(8)**

2. a Find the directional derivative of ϕ=3x^2+2y-3z at (1,1,1) in the direction of $2\vec{i}+2\vec{j}-\vec{k}$ **(8)**

   b Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point (2,-1,2). **(8)**

3. a Find the values of a and b, if the surfaces $ax^2-byz=(a+2)x$ and $4x^2y+z^3=4$ cut orthogonally at the point (1,-1,2). **(8)**

   b Prove that $\vec{F}=\left(y^2\cos x+z^3\right)\vec{i}+(2y\sin x-4)\vec{j}+3xz^2\vec{k}$ is irrotational and find the scalar potential f. **(8)**

4. a Find the work done the force $\vec{F}=3xy\vec{i}-y^3\vec{j}$ moves a particle along the curve C: $y=2x^2$from (0,0) to (1,2) in the xy-plane. **(8)**

   b Evaluate $\int_c \vec{f}.d\vec{r}$ Where $\vec{f}=(2xy+z^3)\vec{i}+x^2\vec{j}+3xz^2\vec{k}$ along the straight line joining $(1,-2,1)$ and (3,2,4) **(8)**

5. a Evaluate $\iint_S \vec{f}.\hat{n}ds$ Where $\vec{f}=(x^2+y^2)\vec{i}-2x\vec{j}+2yz\vec{k}$ and S is the surface of the $2x+y+2z=6$ in the first octant. **(8)**

   b Using Green's theorem in the plane evaluate $\int_C (3x^2-8y^2)dx+(4y-6xy)dy$ Where C is the boundary of the region enclosed by $y=\sqrt{x}$ and $y=x^2$. **(8)**

6. a Apply Green's theorem in the plane to evaluate $\int_C (3x^2-8y^2)dx+(4y-6xy)dy$ Where C is the boundary of the region defined by by $x=0,\ y=0\ \ and\ \ x+y=1.$ **(8)**

b Verify Green's theorem in a plane for $\int (2x - y)dx + (x + y)dy$ where C is the boundary of the circle $x^2+y^2=1$ in the xoy plane **(8)**

7. a Verify Gauss's divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ and C is its boundary over the cube $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1.$ **(8)**

   b Verify Gauss Divergence theorem for $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ Where S is the surface of the cuboid formed by the planes $x = 0, x = a, y = 0, y = b, z = 0, z = c.$ **(8)**

8. a Verify Gauss's divergence theorem for the function $\vec{F} = y\vec{i} + x\vec{j} + z^2\vec{k}$ over the cylindrical region bounded by $x^2 + y^2 = 9,\ z = 0\ and\ z = 2.$ **(8)**

   b Verify stoke's theorem for $\vec{F} = (x^2 - y^2)\vec{i} + 2xy\vec{j}$ in the rectangular region in the xy plane bounded by the lines $x = 0, x = a, y = 0, y = b.$ **(8)**

9. a Verify stoke's theorem for $\vec{F} = (2x - y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$ Where S is the upper half of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary **(8)**

   b Evaluate $\int_C (e^x dx + 2ydy - dz)$.Using stoke's theorem, where C is the curve $x^2 + y^2 = 4, z = 2.$ **(8)**

10. a Verify stoke's theorem for $\vec{F} = -y\vec{i} + 2yz\vec{j} + y^2\vec{k}$. Where S is the upper half of the sphere $x^2 + y^2 + z^2 = 1.$ **(8)**

    b Verify stoke's theorem for $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ taken round the rectangle bounded by the lines $x = \pm a, y = 0\ and\ y = b.$ **(8)**

UNIT- III
ANALYTIC FUNCTIONS

PART-A

1. Is f (z) = z^3analytic ? Justify.
2. Prove that $z\bar{z}$ is nowhere analytic.
3. For what values of a,b and c the function $f(z) = x - 2ay + i(bx - cy)$ is analytic.
4. If u+iv is analytic , show that v –iu &-v +iu are also analytic.
5. State the orthogonal property of an analytical function.
6. Show that the an analytic function with constant real part is constant.
7. Write down the formula for finding an analytic function f (z) = u + iv, whenever the real part is given by using Milne Thomson method.
8. Find 'a' so that u (x,y) = $ax^2 - y^2 + xy$ is harmonic.
9. Verify the function u (x,y) $= \log\sqrt{x^2 + y^2}$ is harmonic or not.
10. Define the conformal mapping.
11. Find the critical points for the transformation $W^2 = (z - \alpha)(z - \beta)$.
12. Find the image of the circle $|z| = 2$ under the information $\omega = 3z$.
13. Find the fixed points of the transformation $w = z^3$.
14. Find the image of $|z - 2i| = 2$ under the mapping w $= \frac{1}{z}$.
15. Define bilinear transformation.

16. Find the fixed point of transformations $\omega = \frac{6z-9}{z}$.

17. Find the fixed points of the transformation w=$\frac{4z-6}{z-1}$

18. Find the fixed points of $w = \frac{(3z-4)}{(z-1)}$.

19. Find the invariant point of the bilinear transformation w =$\frac{1+z}{1-z}$.

20. Write the cross ratio of the points z_1, z_2, z_3, z_4.

PART –B

1. a. If u=$x^2 - y^2$ and v=$\frac{y}{x^2+y^2}$,prove that u and v are harmonic functions but u+iv is not regular function of z. **(8)**

   b.Given that f (z) =u+iv is an analytic function and u+v $= e^x(\cos y + \sin y)$.find f(z). **(8)**

2. a. Prove that an analytic function with constant modulus is constant. **(8)**

   b. If f (z) is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$ **(8)**

3. a. Show that v = e^x (xcosy – ysiny) is a harmonic function . Find the corresponding analytic function f (z). **(8)**

   b. Use Milne Thomson Method to find the harmonic conjugate u of v = e^{-x} (2xycosy +($y^2 - x^2$)siny. Where u+iv is the analytic function. **(8)**

4. a. Show that the v = e^{2x} (ycos2y + x sin2y)is harmonic and find the corresponding analytic function f (z) = u+iv. **(8)**

   b. Find the analytic function whose real part is $\frac{\sin 2x}{(\cosh 2y - \cos 2x)}$ **(8)**

5. a. Prove that the function u =$x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic. Find the conjugate harmonic function V and the corresponding analyticfunction f(z) **(8)**

   b. Find the orthogonal trajectories of the curves represented by **(8)** u(x,y) =$x^3y - xy^3$ = c

6. a. Discuss the conformal mapping W=1/z **(8)**

   b. Find the image of the circle $|z-1|$ = 1 in the complex plane under the mapping $\omega = \frac{1}{z}$ Show the region graphically. **(8)**

7. a. Obtain the image of $|z - 2i| = 2$ under the transformation $\omega = \frac{1}{z}$. **(8)**

   b. Find the image of the circle $|z-1|$ = 1 in the complex plane under the mapping $\omega = \frac{1}{z}$. Show the region graphically. **(8)**

8. a.Find the image of x+y=1 under the transformation w=z^2 **(8)**

   b. Find the bilinear transformations that maps the points z = -1,0,1 in the z – plane on to the points W = 0, i, 3i in the w – plane. **(8)**

9. a. Find the Bilinear transformation that maps the points 1+i, -i, 2-i at the z- plane into the

points 0,1,I of the w-plane. **(8)**

b. Find the bilinear transformation which maps the points z=1, i,-1 into the points w= i, 0,-i. Hence find the image of $|z| < 1$. **(8)**

10.a. Find the bilinear transformation which maps the points z = 0,-I,-1 into w = I,1,0 respectively. **(8)**

b. Find the bilinear transformation which maps -1,0,i of the z-plane onto -1,-I,1 of the w-plane. Show that under this mapping the upper half of the z-plane maps onto the interior of the unit circle $|w| = 1$ **(8)**

UNIT- IV
MULTIPLE INTEGRALS

PART - A

1. Express $\int_0^1 \int_y^{2y} \frac{x^2}{(x^2+y^2)^{3/4}}$ dx dy in polar co-ordinates.
2. Find the limits of integration in the double integral $\iint_R f(x,y)\,dx\,dy$ where R is in the first quadrant and bounded by x =1 , y = 0 , $y^2 = 4x$.
3. Sketch roughly the region of integration for the double integral $\int_0^1 \int_0^x f(x,y)\,dy\,dx$.
4. Shade the region of integration $\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} dx\,dy$
5. Express the region x≥0 , y≥0 , z ≥0 , $x^2+y^2+z^2 \le 1$ by triple integration.
6. Evaluate $\int_0^{\frac{\pi}{2}} \int_0^2 r\,dr\,d\theta$.
7. Find the area of a circle of radius 'a' by double integration in polar co-ordinates.
8. Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} (dy\,dx)$.
9. Evaluate $\int_2^a \int_2^b \frac{dx\,dy}{xy}$
10. Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{\cos\theta} r\,dr\,d\theta$..
11. Express $\int_0^\infty \int_0^\infty f(x,y)\,dx\,dy$ in polar coordinate.
12. Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \sin(x+y)\,dx\,dy$.
13. Draw the region of integration for $\int_0^4 \int_0^{\frac{3\sqrt{16-x^2}}{4}} x\,dx\,dy$.

14. Evaluate $\int_1^2\int_0^y \frac{dxdy}{x^2+y^2}$

15. Find ∬ dx dy over the region bounded by x≥0 , y≥0 , x+y≤1.

16. Evaluate $\int_0^{\frac{\pi}{2}}\int_0^{a\cos\theta} r dr d\theta$

17. Evaluate $\int_4^3\int_1^2 (x+y)^2\, dxdy.$

18. Evaluate $\int_1^2\int_1^3\int_1^2 xy^2 z dz dy dx..$

19. Change the order of integration $\int_0^1\int_{x^2}^{2-x} f(x,y)dydx.$

20. Evaluate $\int_0^a\int_0^b\int_0^c e^{x+y+z} dzdydx.$

PART -B

1. a Evaluate ∬ $(x+y)^2$ dx dy over the area bounded by ellipse $x^2/a^2 + y^2/b^2 = 1$. **(8)**

   b Evaluate $\iint_R xydxdy$ where R is the region bounded by the parapola $y^2 = x$ and the lines y=0 and x+y=2 lying in the first quadrant **(8)**

2. a Evaluate $\int_0^{\frac{\Pi}{2}}\int_0^{\sin\theta} r d\theta dr$ **(8)**

   b Evaluate $\iint r^2 dr d\theta$,over the area bounded between the circles r=2cosθand r=4cosθ.**(8)**

3. a Change the order of integration in $\int_0^a\int_{\frac{x^2}{a}}^{2a-x} xydydx$ and then evaluate it. **(8)**

   b Change the order of integration and hence evaluate $\int_0^1\int_x^1 \frac{x}{(x^2+y^2)}dxdy$ **(8)**

4. a Evaluate $\int_0^3\int_0^{\sqrt{(4-y)}} (x+y)dxdy$ by changing the order of integration. **(8)**

   b Change the order of integration and evaluate the integral $\int_0^a\int_x^a (x^2+y^2)dydx$ **(8)**

5. a Find by double integration of the area between the parabola $y^2 = 4ax$ and the line y = x .

   b Find the area of the region bounded by $y = x^3$, $y = \sqrt{x}$ using double integrals **(8)**

6. a Find the area of the region bounded by the parabolas $y = x^2$ and $x = y^2$. **(8)**

   b Find the area enclosed by the two parabolas $y^2=4x$ and $x^2=4y$ **(8)**

7. a Evaluate $\int_0^a \int_y^a \frac{x}{(x^2+y^2)} dxdy$ by changing into polar co-ordinates. **(8)**

b Transform the integral into polar coordinates and hence evaluate

$\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2}\, dydx..$ **(8)**

8. a By converting into polar co-ordinates evaluate, $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x}{(x^2+y^2)} dxdy$ **(8)**

b Find the volume of the tetrahedron formed by the planes x = 0,y = 0, z= 0 and x+y+z=1. **(8)**

9. a Evaluate $\iiint xyzdxdydz$ taken over the positive octant of the sphere $x^2+y^2+z^2=a^2$. **(8)**

b Evaluate $\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^a x^2 \sin y dxdydz$ **(8)**

10 a. Express the volume of the sphere $x^2+y^2+z^2 = a^2$ as a volume integral and hence evaluate it. **(8)**

b. Evaluate ∭ dx dy dz /$\sqrt{a^2-x^2-y^2-z^2}$ over the first octant of the sphere $x^2+y^2+z=a^2$. **(8)**

UNIT – V
COMPLEX INTEGRATION

PART – A

1. State Cauchy's integral formula.
2. Evaluate $\int_c \frac{1}{2z-3} dz$ where c is the circle $|z|=1$
3. Evaluate $\frac{1}{2\pi i}\int_c \frac{z^2+5}{z-3} dz$ where c is $|z| = 4$ using Cauchy's integral formula.
4. Evaluate $\int_c \frac{z^2+1}{z-1} dz$ where c is a circle $|z|=2$
5. Evaluate $\int_c \frac{\cos \pi z}{z^2-1} dz$ around a rectangle with vertices at 2±i,-2±i
6. Find the Laurent series expansions of $f(z)=\frac{e^{2z}}{(z-1)^3}$ about z = 1.
7. Obtain the expansion of log (1 + z) when $|z| < 1$
8. Expand $\frac{1}{z-2}$ at z = 1 in a Taylors series
9. Expand cos z in a Taylor's series at z = π / 4
10. Obtain the Laurent expansion of the function $\frac{e^z}{(z-1)^2}$ in the neighbourhood of its

11. Evaluate $\int_c \frac{dz}{(z-3)^2}$ where C is circle $|z| = 1$

12. Find the value of $\int_c \frac{e^{-z}dz}{z^2}$

13. Determine the residues at poles of the function $f(z) = \frac{z+4}{(z-1)(z-2)}$

14. Find the residue at the pole of the function $f(z) = \frac{2z+3}{(z+2)^2}$

15. Find the residue of z $e^{1/z}$ at its singular point.

16. Using Cauchy's integral formula evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-2(z-3))}$ dz where c is the circle $|z|$=4

17. State Cauchy's Residue Theorem.

18. Express $\int_0^{2\pi} \frac{d\theta}{1 + a\cos\theta}$ as a contour integral around the circle $|z| = 1$

19. Find the Residue of $f(z) = \frac{z}{(z-1)^2}$ as its pole.

20. Find the residue of cot z at the pole z=0.

PART–B

1.a. Using Cauchy's integral formula evaluate $\int_c \frac{e^z dz}{(z+2)(z+1)^2}$ where c is $|z| = 3$ **(8)**

b. Using Cauchy's integral formula evaluate $\int_c \frac{z+4}{z^2 + 2z + 5} dz$ where c is $|z+1-i| = 2$ **(8)**

2. a. By Cauchy's integral formula evaluate $\int_c \frac{z+1}{z^4 - 4z^3 + 4z^2}$ where c is $|z-2-i| = 2$ **(8)**

b. Using Cauchy's integral formula evaluate $\int_c \frac{z}{(z-1)(z-2)^2} dz$ where c is circle $|z-2| = \frac{1}{2}$ **(8)**

3. a. Expand $f(z) = \frac{z^2-1}{(z+2)(z+3)} dz$ in Taylors series if $|z|$ < 2 **(8)**

b. Expand $\frac{1}{z-2}$ at z =1 in Taylors series. **(8)**

4.a. Find the Laurent series expansion for the function $f(z) = \frac{7z-2}{(z+1)\ (z)(z-2)}$ in the annular region 1< $|z+1| < 3$ **(8)**

b. Expand $\frac{1}{z^2 - 3z + 2}$ in Laurent series valid in the region (i) 0<$|z-1| < 1$ (ii) 1<$|z|$<2 **(8)**

5.a. Obtain the Laurent series expansion for the function $f(z)=\frac{z+3}{z(z^2-z-2)}$ in the region (i) $|z|<1$ (ii) $1<|z|<2$ (iii) $|z|>2$ **(8)**

b. Using residue theorem to evaluate $\int_c \frac{3z^2+z-1}{(z^2-1)(z-3)}dz$ around the circle $|z|=2$ **(8)**

6. a. Evaluate $\int_{|z|=3} \frac{\cos\pi z^2+\sin\pi z^2}{(z+1)(z+2)}dz$ Using Cauchy's residue theorem. **(8)**

b. Evaluate $\int_c \frac{dz}{(z^2+4)^2}$ where c is the circle $|z-i|=2$ using Cauchy's residue theorem **(8)**

7. a. Evaluate $\int_c \frac{e^z}{(z^2+\pi^2)^2}dz$ where c is the circle $|z|=4$ using Cauchy's residue theorem **(8)**

b. Evaluate $\int_0^{2\pi} \frac{d\theta}{5-\sin\theta}$ by contour integration **(8)**

8. a. Using the method of contour integration evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)}dx$ **(8)**

b. valuate $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta}$,a>b>0by contour integration . **(8)**

9.a. Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5-4\cos\theta}d\theta$ by contour integration **(8)**

b. Prove that $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+a^2)(x^2+b^2)}dx=\frac{\pi}{a+b}, a,b>0$ by using the method of contour integration. **(8)**

10. a. By the method of contour integration. Prove that

$$\int_0^{2\pi} \frac{d\theta}{1-2a\cos\vartheta+a^2}=\frac{2\pi}{1-a^2}, if\ 0<a<1$$ **(8)**

b. Prove that $\int_0^{\infty} \frac{x\sin mx}{x^2+a^2}dx=\frac{\pi}{2}e^{-ma}, m>0; a>0$ **(8)**