



KINGS

COLLEGE OF ENGINEERING



DEPARTMENT OF MATHEMATICS
ACADEMIC YEAR 2010-2011 / EVEN SEMESTER
QUESTION BANK

SUBJECT NAME: STATISTICS AND NUMERICAL METHODS

YEAR/SEM: II/IV

UNIT - I
TESTING OF HYPOTHESIS

PART - A(2 Marks)

1. What is the essential difference between confidence limits and tolerance limits?
2. Define Null hypothesis.
3. What do you mean by test of Hypothesis two tailed test?
4. What is Test of significance?
5. Define student's t-test for difference of means of two samples.
6. Write down the formula of test statistic 't' to test the significance of difference between the means.
7. Write the application of t-test?
8. What is the assumption of t-test?
9. State the important properties of 't' distribution.
10. Define χ^2 test of goodness of fit.
11. Define errors in sampling and critical region.
12. Write the application of 'F' test
13. Define a 'F' variate
14. A random sample of 25 cups from a certain coffee dispensing machine yields a mean $\bar{x} = 6.9$ ounces per cup. Use $\alpha = 0.05$ level of significance to test, on the average, the machine dispense $\mu = 7.0$ ounces against the null hypothesis that, on the average, the machine dispenses $\mu < 7.0$ ounces. Assume that the distribution of ounces per cup is normal, and that the variance is the known quantity $\sigma^2 = 0.01$ ounces.
15. In a large city A, 20 percent of a random sample of 900 school boys had a slight physical defect. In another large city B, 18.5 percent of a random sample of 1600 school boys had some defect. Is the difference between the proportions significant?
16. A sample of size 13 gave an estimated population variance of 3.0 while another sample of size 15 gave an estimate of 2.5. Could both samples be from populations with the same variance?
17. Give the main use of χ^2 test.
18. What are the properties of "F" test.
19. Write the condition for the application of χ^2 test.
20. For a 2 x 2 contingency table

a	b
c	d

, write down the corresponding χ^2 value

PART – B(16 Marks)

1. (a) Suppose that 100 litres made by a certain manufacturer lasted on the average 21,819 miles with a standard deviation of 1,295 miles. Test the null hypothesis $\mu=22,000$ miles against the alternative hypothesis $\mu < 22000$ miles at the 0.05 level of significance. **(8)**

(b) In a study designed to investigate whether certain detonators used with explosives in coal mining meet the requirement that atleast 90% will ignite the explosive when charged, it is found that 174 of 200 detonators function properly. Test the null hypothesis $p=0.90$ against the alternative hypothesis $p < .90$ at the 0.05 level of significance. **(8)**

2. (a) A survey of 320 families with five children each revealed the following distribution:

Number of Boys: 0 1 2 3 4 5

Number of Girls: 5 4 3 2 1 0

Number of Families: 12 40 88 110 56 14

Is this result consistent with the hypothesis that male and female births are equally probable? **(8)**

(b) An experiment yielded the following results:
 $n_1=13, s_1^2=19.2$ $n_2=16$ and $s_2^2=3.5$. Test the null hypothesis $\sigma_1^2 = \sigma_2^2$ against the alternative $\sigma_1^2 \neq \sigma_2^2$ at the 0.01 level of significance. **(8)**

3. (a) A commonly prescribed drug for relieving Nervous tension is believed to be only 60% effective. Experimental results with a new drug administered to a random sample of 100 adults who were suffering from nervous tension show that 70 received relief. Is this sufficient evidence to conclude that the new drug is superior to the one commonly prescribed? Use a 0.05 level of significance. **(8)**

(b) Two independent samples of 8 and 7 items respectively had the following values Of the variable (weight in kgs.)

Sample I : 9 11 13 11 15 9 12 14

Sample II: 10 12 10 14 9 8 10

Use 0.05 level of significance to test whether it is reasonable to assume that the variances of the two population's sample are equal. **(8)**

4. (a) Two independent samples of eight and seven items respectively had the following values of the variables.

Sample-1: 9 11 13 11 15 9 12 14

Sample-2: 10 12 10 14 9 8 10

Do the two estimates of population variance differ significantly at 5% level of Significance. **(10)**

(b) Experience has shown that 20% of a manufactured product is of top quality. In one day's production of 400 articles, only 50 are of top quality. Show that either the Production of the day chosen was not a representative sample (or) the hypothesis of 20% was wrong. **(6)**

5. (a) The following are the average weekly losses of working hours due to accidents in 10 industrial plants before and after an introduction of a safety program was put into operation.

Before : 45 73 46 124 33 57 83 34 26 17

After : 36 60 44 119 35 51 77 29 24 11

Use to 0.05 level of significance to test whether the safety program is effective. **(8)**

(b) A study shows that 16 out of 200 tractors produced on one assembly line required extensive adjustments before they could be shipped, while the same was true for 14 out of 400 tractors produced on another assembly line. At the 0.01 level of significance, does this support the claim that the second production line does support work? (8)

6. (a) In a referendum submitted to the students to the body at a University, 850 men and 560 women voted. 500 men and 320 women voted favorably. Does this indicate a significant difference of opinion between men and women on this matter at 1% level of significance. (8)

(b) Two independent samples are chosen from two schools A and School B and a common test is given in a subject. The scores of the students are as follows.

School A :	76	68	70	43	94	68	33	
School B :	40	48	92	85	70	76	68	22

(8)

7. (a) Two independent samples of sizes 8 and 7 contained the following values.

Sample I :	19	17	15	21	16	18	16	14
Sample II :	15	14	15	19	15	18	16	

Test if the two populations have the same variance. (8)

(b) Do the following data gives evidence of the effectiveness of inoculation?

Attacked Not-Attacked

Inoculated	20	300	
Not-Inoculated	80	600	

(8)

8. (a) Two researches adopted different sampling techniques while investigating the same group of students to find the number of students falling into different Intelligence level.

Research	Below Average	Average	Above Average	Excellent	Total
X	86	60	44	10	200
Y	40	33	25	2	100
Total	126	93	69	12	300

Would you say that the sampling techniques adopted by the two researchers are significantly different? (8)

(b) A machinist is making engine parts with axle diameters of 0.7 inch. A random sample of 10 parts shows a mean diameter of 0.742 inch with a standard deviation of 0.04 inch. Compute the statistic you would use to test whether the work is meeting the specification. (8)

9. (a) Two random samples gave the following results:

Samples	Size	Sample mean	Sum of the squares of deviations from the mean
1	10	15	90
2	12	14	108

Examine whether the samples come from the same normal population. (8)

(b) To verify whether a course in accounting improved performance, a similar test was given to 12 participant both before and after the course. The marks are:

Before :	44	40	61	52	32	44	70	41	67	72	53	72
After :	53	38	69	57	46	39	73	48	73	74	60	78

Was the course useful? (8)

10. (a) In an experiment on immunization of cattle from tuberculosis the following results were obtained:

	Attacked	Not-Attacked	
Inoculated	12	26	
Not-Inoculated	16	6	(8)

Calculate χ^2 and discuss the effect of vaccine in controlling susceptibility to tuberculosis.

(b) Suppose we wish to analyze a set of data to determine if success in medical school is related to ability in mathematics. Suppose that 150 randomly selected medical students were rated according to their success in medical school and their ability in mathematics with respect to each of these characteristics the students were rated as low, average or high. The number of students in each category is given in the following table:

Ability in Mathematics

		Low	Average	High
		14	8	5
Success in Medical School	Low	12	51	11
	Average	7	24	18
	High			

On the basis of contingency table, should we conclude that success in medical school is related to ability in mathematics? Test at the 0.05 level of significance. (8)

UNIT – II DESIGN OF EXPERIMENTS

PART - A(2 Marks)

1. Write down the format the ANOVA table for two factors of classification.
2. Write any two differences between RBD and LSD
3. Write the basic assumptions in analysis of variance.
4. Write the advantages of the Latin square design over the other design.
5. What is ANOVA?
6. Why a 2 x 2 Latin square is not possible? Explain.
7. What is the aim of the design of experiments?
8. Define completely randomized design.
9. What do you mean by Latin square design?
10. State the Null and Alternative hypotheses for a completely randomized design.
11. What is the purpose of blocking in a randomized block design?
12. What is main advantage of Latin square Design over Randomized Block Design?
13. What is meant by tolerance limits?

14. When do you apply analysis of variance technique?
15. Write down the ANOVA table for Latin square.
16. Define Replication.
17. Write down the format the ANOVA table for one factors of classification.
18. Write down the format the ANOVA table for three factors of classification.
19. Define Randomisation
20. Define Local control.

PART - B(16 Marks)

1. Analyze the variance in the Latin square of yields (in kgs.) paddy where P,Q,R,S denote the different methods of cultivation

S122	P121	R123	Q122
Q124	R123	P122	S125
P120	Q119	S120	R121
R122	S123	Q121	P122

Examine whether the different methods of cultivation have given significantly different yields

(16)

2. The following data represent the number of units production per day turned out by different workers using 4 different types of machines.

Machine type

	A	B	C	D
1	44	38	47	36
2	46	40	52	43
3	34	36	44	32
4	43	38	46	33
5	38	42	49	39

Test whether the five men differ with respect to mean productivity and test whether the mean productivity is the same for the four different machine types.

(16)

3. The three samples below have been obtained from normal population with equal variance. Test the hypothesis that the sample means are equal.

8	7	12
10	5	9
7	10	13
14	9	12
11	9	14

(16)

4. The following table gives the number of refrigerators sold by 4 salesman in three months:

Month	Salesman			
	A	B	C	D
I	50	40	48	39
II	46	48	50	45
III	39	44	40	39

Is there a significant difference in the sale made by the four salesman ?

Is there a significant difference in the sales made during different month? **(16)**

5. A variable trial was conducted on wheat with four varieties in a Latin Square Design.

The plan of the experiment and the per plot yield are given below;

C(25)	B(23)	A(20)	D(20)
A(19)	D(19)	C(21)	B(18)
B(19)	A(14)	D(17)	C(20)
D(17)	C(20)	B(21)	A(15)

(16)

Analyse the data.

6. A farmer wishes to test the effects of four different fertilizers A,B,C,D on the yield of wheat. In order to eliminate sources of error due to variability in soil fertility, he uses the fertilizers, in a Latin square arrangement as indicated in the following table, where the numbers indicate yields in bushels per unit area.

A	18	C	21	D	25	B	11
D	22	B	12	A	15	C	19
B	15	A	20	C	23	D	24
C	22	D	21	B	10	A	17

Perform an analysis of variance to determine if there is a significant difference between the fertilizers at $\alpha=0.05$ levels of significance. **(16)**

7. An experimental was designed to study the performance of 4 different detergents for cleaning fuel injectors. The following “Cleanness” readings were obtained with specially designed equipment for 12 tanks of gas distributed over 3 different model of engines.

	Engine 1	Engine 2	Engine 3
Detergent A	45	43	51
Detergent B	47	46	52
Detergent C	48	50	55
Detergent D	42	37	49

Looking on the detergents of treatments and the engines at blocks. Obtain the appropriate analysis of variance table and test at the 0.01 level of significance whether there are differences in detergents or in engines. **(16)**

8. A completely randomized design experiment with 10 plots and 3 treatments gave the following results:

Plot No:	1	2	3	4	5	6	7	8	9	10
Treatment	A	B	C	A	C	C	A	B	A	B
Yield	5	4	3	7	5	1	3	4	1	7

Analyse the results for treatment effects. **(16)**

9. Three machines A,B,C gave the production of pieces in four days as below:

A	17	16	14	13
B	15	12	19	18
C	20	8	11	7

Is there a significant difference between machines? **(16)**

10. Four varieties A,B,C,D of a fertilizer are tested in a Randomized Block Design with four replication. The plot yields in pounds are as follows.

A12	D20	C16	B10
D18	A14	B11	C14
B12	C15	D19	A13
C16	B11	A15	D20

(16)

Analyse the experimental yield.

11. A part of the investigation of the collapse of the roof of a building, a testing laboratory is given all the available bolts that connected the steel structure at three different positions on the roof. The forces required to shear each of these bolts (coded values) are as follows:

Position 1: 90,82,79,98,83,91

Position 2: 105,89,93,104,89,95,86

Position 3: 83,89,80,94

Perform an analysis of variance to test at the 0.05 level of significance whether the differences among the sample means at the three positions are significant. **(16)**

12. There are three main brands of a certain powder. A set of 120 sample values is examined and found to be allocated among four groups (A,B,C and D) and three brands (I,II,III) as shown here under.

Brands	Groups			
	A	B	C	D
I	0	4	8	15
II	5	8	13	6
III	8	19	11	13

(16)

Is there any significant difference in brands preference? Answer at 5% level.

13. Perform two-way Analysis of variance for the given data:

Plots of Land	Treatment			
	A	B	C	D
I	38	40	41	39
II	45	42	49	36
III	40	38	42	42

(16)

14. Three varieties of a crop are tested in a randomized block design with four replications, the layout being as given below: The yields are given in kilograms. Analyse for significance.

C48	A51	B52	A49
A47	B49	C52	C51
B49	C53	A49	B50

Rewriting the data such that the rows represent the blocks and the columns represent the varieties of the crop (as assumed in the discussion of analysis of variance for two factors of classification), we have the following table. (16)

15. Four doctors each test four treatments for a certain disease and observe the number of days each patient takes to recover. The results are as follows (recovery time in days)

Doctor	Treatment			
	1	2	3	4
A	10	14	19	20
B	11	15	17	21
C	9	12	16	19
D	8	13	17	20

Discuss the difference between (a) doctors and (b) treatments. (16)

UNIT – III

SOLUTION OF EQUATIONS AND EIGENVALUE PROBLEMS

PART-A(2 Marks)

- 1 Give an example of (a) algebraic (b) transcendental equation.
2. Write the formula for finding $\sqrt[N]{N}$ where N is a real number, by Newton's method.
3. Write Newton's formula to find the cube root of N.
4. What is the rate of convergence in N.R method?
5. State the criterion for the convergence in Newton Raphson method
6. State any two differences between direct and iterative methods for solving system of equations.

7. Write down the condition for the convergence of Gauss – Seidel iteration scheme.
8. Distinguish Gauss Elimination method and Gauss Jordan method.
9. When Gauss-Elimination method fails.
10. Solve by Gauss – Jordan method the following system of equation .
 $2x_1 + x_2 = 3, x_1 + 2x_2 = 3.$
11. Distinguish between direct and iterative (indirect) method of solving simultaneous equations.
12. When will the solution of $AX = B$ by Gauss – Seidel method converge Quickly?
13. What are the two types of errors involving in the numerical computations?
14. Define Round off error and Truncation error
15. Explain the term “Pivot elements”.
16. State the principle used in Gauss-Jordan method.
17. When Gauss Seidal converge.
18. Solve the system of equations $x-2y=0, 2x+y=5$ by Gaussian elimination method
19. Find the dominant eigenvalue of $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ by power method?
20. How will you find the smallest eigen value of a square matrix A?

PART – B(16 Marks)

1. (a). Solve by Newton-Raphson method the equation $x \sin x + \cos x = 0$ correct to four decimal places.. (8)
- (b) Show that Newton-Raphson formula to find \sqrt{a} can be in the form

$$x_{n+1} = \frac{1}{2} \left(x_n + \frac{a}{x_n} \right), a = 0, 1, 2, \dots$$
 (8)
2. (a). Derive a Newton-Raphson iteration formula for finding the cube root of a positive number N. Hence find $\sqrt[3]{12}$ (8)
- (b) Obtain an iteration formula, using N - R values to find the reciprocal of a given number N and hence find $\frac{1}{19}$, correction of 4 decimal places (8)
3. (a) Find the negative root of the equation $x^2 + 4 \sin x = 0$ by Newton-Raphson method correct to three decimal places (8)
- (b). Solve the following system of equation using Gauss – elimination method
 $2x + y + 4z = 12, 8x - 3y + 2z = 20, 4x + 11y - z = 33.$ (8)
4. (a). Solve the system of equations $28x + 4y - z = 32, x + 3y + 10z = 24, 2x + 17y + 4z = 35$ by Gauss elimination method.
- (b) Using Gauss-Jordan method, solve the following system of equations
 $2x - y + 2z = 8, -x + 2y + z = 4, 3x + y - 4z = 0.$ (8)
5. (a). Using Gauss-Jordan method, solve the following system of equations
 $5x + 4y = 15, 3x + 7y = 12$ (8)
- (b). Using Gauss – Seidel method, solve the following system. Start with $x = 1, y = -2, z = 3.$
 $x + 3y + 52z = 173.61; x - 27y + 2z = 71.31; 41x - 2y + 3z = 65.46$ (8)
6. (a) Solve the following equation using Jacobi's iteration method
 $30x - 2y + 3z = 75, x + 17y - 2z = 48, x + y + 9z = 15$ (8)
- (b) Solve the following system of equation using Jacobi's iteration method.
 $8x - 3y + 2z = 20, 6x + 3y + 12z = 35, 4x + 11y - z = 33$ (8)

7. (a). Solve the following system of equation using Gauss – Seidal iterative method
 $2x+10y+z=13$, $10x+y+z=12$, $x+y+5z=7$ (8)

(b) Find the inverse of the given matrix by Gauss Jordan method $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ (8)

8. (a) Using Gauss-Jordon method , find the inverse of the matrix $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$ (8)

(b). Determine the largest eigen value and the corresponding eigen vector correct to 3 decimal places, using power method for the matrix $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ (8)

9. (a). Find the numerically largest eigen values of $A = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$ by power method corresponding eigen vector (correct to 3 decimal places). Start with initial eigen value $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ (8)

(b). Using Power method, obtain the dominant eigen value and corresponding eigen vector correct to two decimal places, for the matrix

$\begin{pmatrix} 2 & 2 & 2 \\ 2/3 & 5/3 & 5/3 \\ 1 & 5/2 & 11/2 \end{pmatrix}$ taking $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ as the initial approximation to the eigen vector. (8)

10. (a). Using power method, find the dominant eigen values and the

corresponding eigen vector of $A = \begin{pmatrix} \frac{1}{\sqrt{2}} & 1 & \sqrt{2} \\ 1 & \frac{3}{\sqrt{2}} & 1 \\ \sqrt{2} & 1 & \frac{1}{\sqrt{2}} \end{pmatrix}$ (8)

(b). Find the largest eigen value and eigen vector of the matrix by power

method $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$ (8)

UNIT IV

INTERPOLATION, NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION

PART- A (2 Marks)

1. Explain the use of Lagrange's interpolation formula.
2. Find the polynomial which takes the following values
Given $f(0) = -1$, $f(1) = 1$ and $f(2) = 4$. Find the Newton's interpolating formula.
3. State any two properties of divided differences.
4. If $f(x) = (1/x^2)$ find $f(a,b)$ and $f(a,b,c)$ by using divided differences.
5. When will we use Newton's forward interpolation formula
6. When will we use Newton's backward interpolation formula.
7. In numerical integration, what should be the number of intervals to apply Simpson's 1/3 rule and by Simpson's 3/8 rule?
8. What are the n^{th} divided differences of a polynomial of the n^{th} degree.
9. State Newton – Gregory forward difference interpolation formula.
10. Find the divided difference for the following data:
11. Find the polynomial for the following data by Newton's backward difference formula:

X:	0	1	2	3
F(x)	-3	2	9	18

12. Find the value of Y at $x = 21$ using Newton's forward difference formula from the following table:

X:	20	23	26	29
Y:	0.3420	0.3907	0.4384	0.4848

13. Find $f(2.5)$ from the data:

X:	1	2	3
F(x)	0	1	8

14. Find the sixth term in the sequence 8,12,19,29,42,.....
15. State the formula for Trapezoidal rule of integration
16. Using Trapezoidal rule evaluate $\int_0^{\pi} \sin x dx$ by dividing the range into 6 equal parts.
17. State Simpson's 1/3 rule formula to evaluate $\int_a^b f(x) dx$.
18. What is the order of error in Simpson's formula?
19. What is the local error term in Trapezoidal formula and in Simpson's one third rule?
20. Compare Trapezoidal rule and Simpson's 1/3 rule for evaluating numerical integration

PART - B(16 Marks)

1. (a). Find the Lagrange's polynomial of degree 3 to fit the data: (8)
 $Y(0)=-12, y(1)=0, y(3)=6$ and $y(4) =12$. Hence find $y(2)$.

(b). Using Lagrange's formula, fit a polynomial to the data (8)
 X: 0 1 3 4
 Y: -12 0 6 12 Also find y at $x=2$.

2. (a). Using Lagrange's formula, fit a polynomial to the data (8)
 X: 0 1 2 4 5 6
 F(x): 1 14 15 5 6 19 Also find $f(3)$.

(b). Using Lagrange's formula, fit a polynomial to the data (8)

X:	-1	0	2	3
Y:	-8	3	1	12

Hence find y at $x=1.5$ and $x=1$

3. (a). If $f(0) = 0, f(1) = 0, f(2) = -12, f(4) = 0, f(5) = 600$ and $f(7) = 7308$, find a polynomial that satisfies this data using Newton's divided difference interpolation formula., Hence, find $f(6)$ (8)

(b). Using Newton's divided difference formula find $f(x)$ and $f(6)$ from the following data: (8)

X:	1	2	7	8
f(x)	1	5	5	4

4. (a). Using Newton's divided difference formula find the value of $f(8)$ and $f(6)$ from the following data: (8)

X:	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

Also find $f(-2)$ and $f(12)$.

(b). Using Newton's divided difference formula find the cubic function of x from the following data: (8)

X:	0	1	4	5
f(x)	8	11	68	123

5. (a). Using Newton's divided difference formula find the cubic function of x from the following data: (8)

X:	0	1	4	5
f(x)	2	3	12	147

(b). Find the polynomial of degree two for the data by Newton's forward difference method: (8)

X:	0	1	2	3	4	5	6	7
F(x)	1	2	4	7	11	16	22	29

6. (a). From the data given below, find the number of students whose weight is between 60 lbs to 70 lbs
 Weight in lbs : 0-40 40-60 60-80 80-100 100-120
 Number of students: 250 120 100 70 50 (8)

- (b). From the following table, find the value of $\tan 45^\circ 15'$ by Newton's forward interpolation formula (8)

X°:	45	46	47	48	49	50
tanx°	1.00000	1.03553	1.07237	1.11061	1.15037	1.19175

7. (a) Given

X:	1	2	3	4	5	6	7	8
f(x):	1	8	27	64	125	216	343	512

Estimate $f(7.5)$. Use Newton's formula. (8)

- (b). Construct Newton's forward interpolating polynomial for the following data:

X:	4	6	8	10
Y:	1	3	8	16

8. (a) Use Simpson's $\frac{1}{3}$ rd rule to estimate the value of $\int_1^5 f(x)dx$ given

x	:	1	2	3	4	5
f(x)	:	13	50	70	80	100

(8)

- (b) Evaluate $\int_0^2 \frac{dx}{x^2 + x + 1}$ correct to three decimals dividing the range of integration into 8 equal parts using Simpson's $\frac{1}{3}$ rd rule. (8)

9. (a) Find $\frac{dy}{dx}$ at $x = 1.5$ given

x :	1	2	3	4	5
y :	77	78	127	248	375

(8)

- (b). Find the first and second derivatives of y w. r. to x at $x = 10$

x :	3	5	7	9	11
y :	31	43	57	41	27

(8)

10. (a) A river is 80 mts wide. The depth 'd' in mts at a distance x mts from one bank is given by the following table. Calculate the area of cross section of the river using

Simpson's $\frac{1}{3}$ rd rule

x :	0	10	20	30	40	50	60	70	80
d :	0	4	7	9	12	15	14	8	3

(8)

- b) The table given below reveals the velocity v of a body during the time 't' specified. Find its acceleration at $t = 1.1$

t :	1.0	1.1	1.2	1.3	1.4
v :	43.1	47.7	52.1	56.4	60.8

(8)

UNIT – V

NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS

PART – A(2 Marks)

1. State the disadvantage of Taylor series methods?
2. Write down the fourth order Taylor Algorithm?
3. Write the merits and demerits of the Taylor method of solution?
4. Which is better Taylor's method or R.K. method?
5. State Taylor series algorithm for the first order differential equation?
6. Solve the differential equation $\frac{dy}{dx} = x + y + xy, y(0) = 1$ by Taylor series method to get the value of y at $x = h$?
7. What is the truncation error in Taylor's series?
8. Write down Euler algorithm to the differential equation $\frac{dy}{dx} = f(x, y)$.
9. State modified Euler algorithm to solve $y' = f(x, y), y(x_0) = y_0$ at $x = x_0 + h$.
10. Using Modified Euler's method, find $y(0,1)$ if $\frac{dy}{dx} = x^2 + y^2, y(0) = 1$
11. What is the error of Euler's method?
12. What is the Error in modified Euler's method?
13. What are the limitations of Euler's method?
14. Write the Runge-Kutta algorithm of second order for solving $y' = f(x, y), y(x_0) = y_0$
15. Write down the Runge-Kutta formula of fourth order to solve $dy/dx = f(x, y)$ with $y(x_0) = y_0$
16. What are the advantages of R.K. method over Taylor method?
(or)
Why is R.K. method preferred to Taylor series method?
(or)
Compare Taylor's series and R.K. method?
17. Write Milne's predictor corrector formula?
18. What is the error of Milne's method?
19. Write the finite difference equivalent of the differential equation $y'' + 2y' + y = x^2$
20. State the conditions for the equation.
 $Au_{xx} + Bu_{yy} + Cu_{xy} + Du_x + Eu_y + Fu = G$ where A, B, C, D, E, F, G are function of x and y to be (i) elliptic (ii) parabolic (iii) hyperbolic

PART – B(16 Marks)

1. a) Using Taylor series method find y at $x = 0.1$ if $\frac{dy}{dx} = x^2 y - 1, y(0) = 1$. (8)
- b) Solve $\frac{dy}{dx} = x^2 + y^2, y(0) = 1$. Use Taylor series at $x=0.2$ and 0.4 , Find y at $x = 0.1$. (8)

2. a) Using Taylor series method find y at $x = 0.1$ correct to four decimal places from $\frac{dy}{dx} = x^2 - y, y(0) = 1$. with $h = 0.1$. Compute terms upto x^4 . (8)
- b) Solve $y' = x + y, y(1) = 0$, by Taylor's series method. Find $y(1.1)$. (8)
3. a) Find the Taylor series solution with three terms for the initial value problem $y' = x^3 + y, y(1) = 1$. (8)
- b) Using Taylor series method with the first five terms in the expansion find $y(0.1)$ correct to three decimal places, given that $y' = e^x - y^2, y(0) = 1$. (8)
4. a) Using Taylor series method, find $y(1.1)$ and $y(1.2)$ correct to four decimal places given. $\frac{dy}{dx} = xy^{1/3}$ and $y(1) = 1$. (8)
- b) Using Euler's method find $y(0.2)$ and $y(0.4)$ from $y' = x + y, y(0) = 1$, with $h = 0.2$. (8)
5. a) Using Euler's improved method find $y(0.1)$ from $y' = x + y + xy, y(0) = 1$, with $h = 0.05$. (8)
- b) Using Euler's method find $y(0.3)$ if $y(x)$ satisfies the initial value problem. $y' = \frac{1}{2}(x^2 + 1)y^2, y(0.2) = 1.1114$, with $h = 0.2$ (8)
6. a) Using modified Euler's method, compute $y(0.1)$ with $h = 0.1$ from $\frac{dy}{dx} = y - \frac{2x}{y}, y(0) = 1$. (8)
- b) Solve $y' = 1 - y, y(0) = 0$, by improved Euler's method. (8)
7. a) Given $\frac{dy}{dx} = x^3 + y, y(0) = 2$. Compute $y(0.2), y(0.4)$ and $y(0.6)$ by Runge-Kutta method of fourth order. (8)
- b) Using R.K. method of 4th order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}, y(0) = 1$. at $x = 0.2$. (8)
8. Consider the initial value problem $\frac{dy}{dx} = y - x^2 + 1, y(0) = 0.5$. (16)
- (i) Using the modified Euler method, find $y(0.2)$
- (ii) Using 4th order Runge-Kutta method, find $y(0.4)$ and $y(0.6)$ Using Adam-Bashforth Predictor- Corrector method. Find $y(0.8)$.
9. (a) Find $y(0.8)$ given that $y' = y - x^2, y(0.6) = 1.7379$ by using Runge-Kutta method of order 4 with $h = 0.1$. (8)
- (b) Using Milne's method find $y(2)$ if $y(x)$ is the solution of $\frac{dy}{dx} = \frac{1}{2}(x + y)$. given $y(0) = 2, y(0.5) = 2.636, y(1) = 3.595$ and $y(1.5) = 4.968$ (8)
10. (a) Given $\frac{dy}{dx} = x^3 + y, y(0) = 2$. The values of $y(0.2) = 2.073, y(0.4) = 2.452$ and $y(0.6) = 3.023$ are got by R.K. method of fourth order. Find $y(0.8)$ by Milne's predictor-corrector method taking $h = 0.2$. (8)
- (b) Solve by finite difference method, the equation $y'' - y = 0$, given $y(0) = 0, y(1) = 1$ with $h = .2$