KINGS
COLLEGE OF ENGINEERING

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

QUESTION BANK

SUBJECT CODE : MA1251 YEAR / SEM: V / III

SUBJECT NAME : NUMERICAL METHODS

UNIT I

SOLUTION OF EQUATIONS & EIGEN VALUE PROBLEMS

PART – A (2 MARKS)

1. If f(x) has not between $x = a$ and $x = b$, then write the first approximate root by the method of false position .
2. Explain method of false position for finding roots of $f(x) = 0$
3. Using N – R method find the root between 0 and 1 of $x^3 = 6x - 4$ correct to 3 decimal places.
4. Explain geometrically N – R method to find a root of $f(x) = 0$ and hence derive a formula for it.
5. Define a Newton-Raphson iteration formula for finding the cube root of a positive number N. Hence find $3\sqrt{12}$
6. What is the criterion for the convergence of N-R method?
7. Find a root of $f(x) = xe^x - \cos x$ lying in the interval (0,1) by N-R method.
8. Obtain the Newton-Raphson iterative formula, for approximating the cube-root of a positive number N, and hence calculate $(23)^{1/3}$ correct to three –decimal places.
9. What is the sufficient condition for the convergence of a root of $f(x) = 0$ by Newton-Raphson method.

10. Solve the following system of equations 2x +y = 3 , 7x – 3y = 4 by Gauss elimination method.
11. Solve the system of equations x - 2y = 0 , 2x + y = 5 by Gaussian elimination method.
12. Solve using Gauss – Jordon method 11x + 3y = 17 , 2x +7y =16.
13. Solve by Gaussian Elimination method x + 2y + z = 3 , 2x + 3y + 3z = 10,
14. 3x – y + 2z = 13.
15. Compare Gaussian elimination and Gauss-Jordan methods in solving the linear system [A]{X}={B}.
16. Using Gaussian elimination meothd represent the system of equations $\begin{pmatrix} 1 & 2 & 1 \\ 2 & -3 & -1 \\ 3 & 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ 3 \end{pmatrix}$ in the upper triangular form.
17. Explain the different between Gauss – Jacobi and Gauss-seidal iterative method
18. State the sufficient condition for convergence of Gauss-seidal method.
19. What is meant by iterative method of solution of an equation? Give an example.
20. Write down the condition for the convergence of Gauss seidal iterative method.
21. By using Jacobi iterative method find the third approximation $\left(x^{(3)}, y^{(3)}, z^{(3)}\right)$ to the solution of the following system the initial approximation being $x^{(0)} = y^{(0)} = z^{(0)}$ 8x + y + z = 8 , 2x + 4y +z = 4 , x + 2y + 4z = 5.
22. What is the condition on A for solving the system of equation AX = B by Jacobi and Gauss-seidel iterative methods ?
23. State the sufficient condition for convergence of Gauss-Jacobi method
24. Find the inverse of $\begin{pmatrix} 1 & 2 \\ 2 & -3 \end{pmatrix}$ by Gauss Jordon method.
25. Find the inverse of the matrix by Jordon method.
26. Define Eigen value and Eigen vecor.
27. Find the eigen value of $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ by Power method.

PART –B

1. Determine the root of $xe^x - 3 = 0$, correct to three decimal places using the method of false positions. ` **(8)**
2. Find the real root of the equation $X\log_{10} X - 1.2 = 0$ correct to four places of decimal using false position method. **(8)**
3. Using Newton-Raphson method, find correct to 3 decimal places, the root between 0 and 1 of the equation $x^3 - 6x + 4 = 0$ **(8)**
4. By Newton's iteration method, find the real root of $X \log_{10} X = 1.2$ correct to 3 decimal places. **(8)**
5. Solve by Newton- Raphson method, the system of equation $x^2 + 3x - y - 1 = 0; xy + 3y + 9 = 0$ starting with the approximation $x_0 = -4, y_0 = 6$ **(8)**
6. Find the N – R method , the root of the equation $e^x = 4x$, which is near to 2, correct to the decimal places. **(8)**
7. Find the iteration formula to find $\sqrt{N}$ where N is a positive integer by Newton's method and hence find $\sqrt{11}$ **(8)**
8. Derive a Newton-Raphson iteration formula for finding the cube root of a positive number N. Hence find $\sqrt[3]{12}$ **(8)**
9. Show that Newton-Raphson formula to find $\sqrt{a}$ can be in the form $x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$, a = 0,1,2,… **(8)**
10. Find the negative root of the equation $x^2 + 4\sin x = 0$ by Newton-Raphson method correct to three decimal places. **(8)**
11. Find the real root of following equations by Newton-Raphson method $y^2 = 4ax$. Obtain an iteration formula, using N - R values to find the reciprocal of a given number N and hence find $\frac{1}{9}$, correction of 4 decimal places. **(8)**

12. Obtain an iteration formula, using N - R values to find the reciprocal of a given number N and hence find $\frac{1}{19}$, correction of 4 decimal places. **(8)**

13. Apply Gauss-Jordon method to find solution of the following system
    a. 10x + y + z = 12 , 2x +10y +z = 13 , x + y + 5z = 7 **(8)**

14. Solve by Gauss – Jordon method 2x – 3y +z = -1 , x + 4y +5z =25 ,
    i. 3x – 4y + z =2 correct to 3 decimal places. **(8)**

15. Solve the following system of equation using Gauss – elimination method
    a. 2x + y + 4z = 12 , 8x -3y +2z = 20 , 4x +11y – z =33. **(8)**

16. Using Gauss-Jordan method, solve the following system of equations
    a. 2x –y +2z = 8 , -x +2y + z = 4, 3x + y -4z = 0 . **(8)**

17. Find the inverse of the given matrix by Gauss Jordon method

A= $\begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$) **(8)**

18. Using Gauss-Jordon method , find the inverse of the matrix $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$ **(8)**

19. Using Power method, obtain the largest eigen value and corresponding eigen vector of the matrix $\begin{pmatrix} 1 & 2 \\ 5 & 4 \end{pmatrix}$ **(8)**

20. Determine the largest eigen value and the corresponding eigen vector correct to 3 decimal places, using power method for the matrix A = $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ **(8)**

21. Obtain the power method the numerically largest eigen value of the matrix **(8)**

22. Find the numerically largest eigen values of A = $\begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$ by power method corresponding eigen vector (correct to 3 decimal places). Start with initial eigen value $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ **(8)**

23. Using power method, find the dominant eigen values and the corresponding eigen vector of A = $\begin{pmatrix} \frac{1}{\sqrt{2}} & 1 & \sqrt{2} \\ 1 & \frac{3}{\sqrt{2}} & 1 \\ \sqrt{2} & 1 & \frac{1}{\sqrt{2}} \end{pmatrix}$ **(8)**

24. Find the largest eigen value and eigen vector of the matrix by power method $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$ **(8)**

25. Using Power method, obtain the dominant eigen value and corresponding eigen vector of the matrix $\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ **(8)**

UNIT II – INTERPOLATION & APPROXIMATION

Part -A

1. Using Lagrange's interpolation, find the polynomial through (0,0),(1,1) and (2,2).
2. What do you understand by inverse interpolation?
3. Obtain the Lagrange's interpolating polynomial for the observed data of points (1,1),(2,1) and (3,-2).
4. Write the Cubic Spline formula.
5. State Newton's backward difference interpolation formula.
    a. Write down Newton's forward and backward difference formula.
    b. Find the polynomial which takes the following values

X:	0	1	2
Y:	1	2	1

6. Find the polynomial for the following data by Newton's backward difference formula:

X:	0	1	2	3
F x):	-3	2	9	18

7. Find the value of Y at x =21 using Newton's forward difference formula from the following table:

X:	20	23	26	29
Y:	0.3420	0.3907	0.4384	0.4848

8. Find the polynomial of second degree passing through the points (0,-1) (1,-1) and (2,1) by using Gregory –Newton backward difference formula.

Part – B

(Each question carries 8 marks)

1. Find the Lagrange's polynomial of degree 3 to fit the data: **(8)**
   Y(0)=-12, y(1)=0,y(3)=6 and y(4) =12. Hence find y(2).
2. Using Lagrange's formula, fit a polynomial to the data **(8)**
   X: 0 1 3 4
   Y: -12 0 6 12 Also find y at x=2.
3. Using Lagrange's formula, fit a polynomial to the data **(8)**
   X: 0 1 2 4 5 6
   F(x): 1 14 15 5 6 19 Also find f(3).
4. Using Lagrange's formula, fit a polynomial to the data **(8)**

X:	-1	0	2	3
Y:	-8	3	1	12

   Hence find y at x=1.5 and x=1
5. Using Lagrange's formula, fit a polynomial to the data **(8)**

X:	0	1	3	4
Y:	-12	0	6	12

   Hence find y at x=2.
6. Using Lagrange's formula, fit a polynomial to the data **(8)**

X:	2	5	7	10	121
Y:	18	180	448	1210	2028

   Hence find y at x=6.
7. If f(0) = 0 , f(1) = 0, f(2) = -12, f(4) =(0), f(5) =600 and f(7) = 7308,find a polynomial that satisfies this data using Newton's divided difference interpolation formula., Hence, find f(6) **(8)**

8. Using Newton's divided difference formula find f(x) and f(6) from the following data: **(8)**

X:	1	2	7	8
f(x)	1	5	5	4

9. Using Newton's divided difference formula find the value of f(8) and f(6) from the following data: **(8)**

X:	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

Also find f(-2) and f(12).

10. Using Newton's divided difference formula find the cubic function of x from the following data: **(8)**

X:	0	1	4	5
f(x)	8	11	68	123

11. Using Newton's divided difference formula find the cubic function of x from the following data: **(8)**

X:	0	1	4	5
f(x)	2	3	12	147

12. Given the following table ,find f(2.5) using cubic spline functions: **(8)**

I:	0	1	2	3
Xi:	2	3	12	147
f(Xi):	0.5	0.3333	0.25	0.2

13. The following values of X and Y are given: **(8)**

X:	1	2	3	4
Y:	1	2	5	11

Find the cubic splines and evaluate Y(1.5).

14. Find the cubic splines for the data : **(8)**

X:	0	1	2	3
f(x):	1	2	9	28

15. Find the cubic splines for the data : **(8)**

X:	1	2	3
f(x):	-6	-1	16

16. Find the polynomial of degree two for the data by Newton's forward difference method: **(8)**

X:	0	1	2	3	4	5	6	7
F x):	1	2	4	7	11	16	22	29

17. From the following table of half-yearly premium for policies maturing at different ages, estimate the premium for policies maturing at age 46 and 63. **(8)**

AgeX:	45	50	55	60	65
PremiumY:	114.84	96.16	83.32	74.48	68.48

18. From the following table, find the value of tan 45◦15' by Newton's forward interpolation formula **(8)**

X◦:	45	46	47	48	49	50
tanx◦	1.00000	1.03553	1.07237	1.11061	1.15037	1.19175

19. Given **(8)**

X:	1	2	3	4	5	6	7	8
f x):	1	8	27	64	125	216	343s	512

Estimate f(7.5) . Use Newton's formula.

20. Construct Newton's forward interpolating polynomial for the following data: **(8)**

X:	4	6	8	10
Y:	1	3	8	16

21. From the following data evaluate y(142) andY(175) **(8)**

X:	140	150	160	170	180
Y:	3.685	4.854	6.302	8.076	10.225

22. Find the cubic polynomial which takes the following values: **(8)**

X:	0	1	2	3
Y:	1	2	1	10

23. Find the value of y at x=1.05 from the table given below **(8)**

X:	1.0	1.1	1.2	1.3	1.4	1.5
Y:	0.841	0.891	0.932	0.964	0.985	1.015

NIT III – NUMERICAL DIFFERENTIATION & INTEGRATION

PART - A

1. Write Newton's forward difference formula to find the derivatives

   $\left(\frac{dy}{dx}\right)_{x=x_0}$ and $\left(\frac{d^2y}{dx^2}\right)_{x=x_0}$

2. State the Bessel's central difference formula.
3. Given that

x :	1.1	1.2	1.3	1.4	1.5	1.6
y :	8.403	8.781	9.129	9.451	9.750	10.031

   find $\frac{dy}{dx}$ at x =1.1.

4. Find y'(x) if

x :	1	2	3	4
y:	1	2	5	13

5. From the following data, find the rate of change of p with respect to v when v=2

V:	2	4	6	8	10
P:	1.5	42.7	25.3	16.7	13

6. Find the second divided differences with arguments a,b,c if $f(x)=\frac{1}{x}$
7. Find f'(4) if

x :	1	2	3	4
y :	1	2	5	13

8. State for formula of Simpson's $\frac{3}{8}$ th rule.
9. Compare Trapezoidal rule and Simpson's $\frac{1}{3}$ rd rule for evaluating numerical integration.
10. Evaluate $\int_1^4 f(x)dx.$ from the table by Simpson's $\frac{3}{8}$ th rule

x :	1	2	3	4

$f(x)$: 1 8 27 64

11. What is the error involved in the Trapezoidal rule?

12. From the following table evaluate $\int_{7.47}^{7.52} ydx.$, using Trapezoidal rule.

x :	7.47	7.48	7.49	7.50	7.51	7.52
y :	1.93	1.95	1.98	2.01	2.03	2.06

13. Evaluate $\int_0^1 (e^x + e^{-x})dx.$, using Simpson's $\frac{1}{3}$rd rule by dividing the interval into four equal parts.

14. Evaluate $\int_0^6 f(x)dx.$ using Trapezoidal rule given the following :

x ;	0	1	2	3	4	5	6
$f(x)$:	1	0.5	0.2	0.1	0.0588	0.0385	0.027

15. Find the value of $\int_1^2 \frac{dx}{x}$ by Simpson's $\frac{1}{3}$rd rule by taking h = ¼ Hence obtain the approximate value of $\log_e 2$.

16. State Trapezoidal rule.

17. Evaluate $\int_{-3}^{3} x^4 dx$ using Trapezodial rule.

18. State two point Gaussian Quadrature Formula to evaluate $\int_{-1}^{1} f(x)d(x).$

19. Evaluate $I = \int_0^1 \frac{dt}{1+t}$ by using three point Gaussian quadrature formula.

20. Write down two-point Gaussian formula of numerical integration.

PART – B

(Each question carries 8 Marks)

1. From the following table, find the value of x for which $f(x)$ is maximum. Also find the maximum value. **(8)**

x : 60 75 90 105 120

$f(x)$: 28.2 38.2 43.2 40.9 37.7

2. The velocity of a train which starts from rest is given by the following table, time being reckoned in minutes from the start and speed miles per hour: **(8)**

Minutes :	2	4	6	8	10	12	14	16	18	20
Miles per hour :	10	18	25	29	321	20	11	5	2	9

Estimate approximately the total distance run in 20 minutes.

3. Find the first and second derivative of the function $f(x) = x^3 - 8x - 14$ at x = 3.0 using the value given below: **(8)**

x:	3.0	3.2	3.4	3.6	3.8	4
$f(x)$:	-14	-10.03	-5.296	-0.256	-6.672	14

4. Given the following pairs of values of x and y **(8)**

x :	1	2	4	8	10
y :	0	1	5	21	27

determine y'(4) using Newton's divided differences.

5. Evaluate $\int_1^2 \int_1^2 \frac{dxdy}{x+y}$ with h=k=0.2 by using trapezoidal rule. **(8)**

6. For the tabulated function : **(8)**

x :	1.0	1.1	1.2	1.3	1.4
y:	0.2500	0.2268	0.2066	0.1890	0.1736]

find y(1.35).y'(1.35) and y"(1.35)

7. Find the value of cos(1.74) using the value given in the following table below: **(8)**

x :	1.70	1.74	1.78	1.82	1.86
sin x :	0.9916	0.9857	0.9781	0.9681	0.9584

8. Find the derivative of $\sqrt{x}$ at x = 1 by formatting the forward **(8)**
difference table the values of $\sqrt{x}$ at x = 1.00, 1.05, 1.10 1.15.

9. From the following table of values of x and y, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

(8)

At x = 1.25

x :	1.00	1.05	1.10	1.15	1.20	1.25

y : 1.00000 1.02470 1.04881 1.07238 1.09544 1.11803

10. Given **(8)**

x:	50	51	52	53	54	55	56
y:	3.6840	3.7084	3.7325	3.7563	3.7598	3.8030	3.8259

Find y' and y" at x = 56.

11. Using Newton's method's find f' at x =1.2 from **(8)**

x:	1.0	1.5	2.0	2.5	3.0
y:	27.00	106.75	324.00	783.75	1621.00

12. By dividing the range into equal parts, evaluate $\int_0^{\pi} \sin x dx$ by using Simpson's $\frac{1}{3}$rd rule.

Is it possible o evaluate he same b impson's $\frac{3}{8}$ h rule. Justify your answer.

(8)

13. Using Romberg's rule, evaluate $\int_0^1 \frac{dx}{1+x}$ correct to there decimal places by taking h=0.5,0.25 and 0.125. **(8)**

14. Use Simpson's $\frac{1}{3}$ rd rule to estimate the value of $\int_1^5 f(x)dx$ given

(8)

x	: 1	2	3	4	5
$f(x)$	: 13	50	70	80	100

15. Compute $\int_0^1 \frac{dx}{1+x^2}$ by using Trapezoidal rule, taking h = 0.5 and h=0.25. Hence find the value of the above integration by Romberg's method. **(8)**

16. Evaluate $\int_0^1\int_0^1 \frac{dxdy}{1+xy}$ using simpson's 1/3 rule, taking h=k=0.5 **(8)**

17. Evaluate $\int_1^2\int_1^2 \frac{dxdy}{x+y}$,taking stepsite h=k=0.25 and using **(8)**
    a. Simpson's 1/3 rule in both directions.
    b. Trapezoidal rule in both directions.

18. Find the value of the following integral using Gaussian quadrature technique $\int_3^5 \frac{4}{(2x^2)}dx$

(8)

19. Use Simpson's $\frac{1}{3}$ rd rule., to integrate sin x between 0 and $\frac{\pi}{2}$ from the following table

(8)

x :	0	$\frac{\pi}{12}$	$\frac{2\pi}{12}$	$\frac{3\pi}{12}$	$\frac{4\pi}{12}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$
sin x :	.00000	.25882	.50000	.70711	.86603	.96593	1.0000

20. Evaluate $\int_0^1 \frac{x}{1+x}dx$ by Simpson's 1/3 rule with h =0.1 **(8)**

21. Using Romberg's method compute $\int_0^1 \frac{dx}{1+x^2}$ by taking h=0.5,0.25.0.125**.**

(8)

22. Evaluate I = $\int_{-1}^1 \frac{dx}{1+x^2}$ by Three point Gaussian Quadrature formula.**(8)**

23. E valuate $I = \int_0^1 \frac{dx}{1+x}$ two point two point Gaussian Quadrature **(8)**

24. Evaluate $I = \int_{-1}^1 \frac{x^2}{1+x^4} dx$ by three point Gaussian formula. **(8)**

25. The function f(x,y) is defined by the following table: **(8)**

x y	0	0.5	1.0	1.5	2.0
1	2.0	1.5	1.3	1.4	1.6
2	3.1	2.5	2.0	2.3	2.9
3	4.2	4.0	3.8	4.1	4.4

Compute $\int_0^3 \int_0^2 f(x,y) dx dy$ using Simpson's rule in the both direct method.

UNIT IV - INITIAL VALUE PROBLEMS FOR ORDINARY DIFFERENTIAL EQUATIONS

PART – A

1. State the disadvantage of Taylor series method.
2. Write down the fourth order Taylor Algorithm.
3. Write the merits and demerits of the Taylor method of solution.
4. Which is better Taylor's method or R,K, Method?
5. State Taylor series algorithm for the first order differential equation.
6. Solve the differential equation $\frac{dy}{dx} = x + y + xy,\ y(0) = 1$ by Taylor series method to get the value of y at x = h.
7. What is the truncation error in Taylor's series?
8. Name the method which is Taylor's method of first order.
9. What is meant by initial value problem and give an example for it.
10. Write down Euler algorithm to the differential equation $\frac{dy}{dx} = f(x, y).$
11. State modified Euler algorithm to solve $\frac{dy}{dx} = f(x, y).$, $y(x_0) = y_0$ at x = x_0 +h.

12. Using Modified Euler's method , find y(0.1) if $\frac{dy}{dx} = x^2 + y^2, y(0) = 1.$
13. What is the error of Euler's method.
14. What are the limitations of Eulers method?
15. What is the Error in modified Euler's method.
16. Write the Runge-Kutta algorithm of second order for solving $\frac{dy}{dx} = f(x, y).$, y $(x_0) = y_0$.
17. State the third order R.K.method algorithm to find the numerical solution of the first order differential equation.
18. Write down the Runge-Kutta formula of fourth order to solve $\frac{dy}{dx} = f(x, y).$ y $(x_0) = y_0$.
19. State the special advantage of Runge-Kutta method over Taylor series method.
20. Is Euler's method formula, a particular case of second order Runge-Kutta method?
21. Compare Taylor's series and R.K. method
22. In the deviation of fourth order Runge-Kutta formula, why it is called fourth order.
23. Write down the formula to solve 2nd order differential equation using Runge-Kutta method of 4th order.
24. What are the values of $k_1 and l_1 y'' + xy' = y, y(0) = 1, y'(0) = 0$ by Runge-Kutta method of fourth order.
25. Write Milne's predictor corrector formula
26. How many prior values are required to predict the next value in Milne's method?
27. What is the error term in Milne's predictor and corrector formula?
28. Write down Adams-Bashforth predictor formula.
29. How many prior values are required to predict the next value in Adam's method?
30. What will you do, if there is a considerable difference between predicted value and corrected value, in predictor corrector methods?
31. Compare Runge-Kutta methods and Predictor – Corrector methods for solution of initial value problem.
32. What is a Predictor-Collector method of solving a differential equation.?
33. What is the condition to apply Adams Bashforth method.
34. What do we mean by saying that a method is self-starting ? Notself starting.

35. Compare the Milne's predictor –Corrector and Adam- Bashforth predictor-corrector methods for solving ordinary differential equations.

PART – B

1. Using Taylor series method find y at x = 0.1 if $\frac{dy}{dx} = x^2 y - 1, y(0) = 1.$ **(8)**
2. Solve $\frac{dy}{dx} = x^2 + y^2, y(0) = 1.$ Use Taylor series at x=0.2 and 0.4, Find x = 0.1. **(8)**
3. Using Taylor series method find y at x = 0.1 correct to four decimal places from $\frac{dy}{dx} = x^2 - y, y(0) = 1.$ with h = 0.1. Compute terms upto x^4. **(8)**
4. Solve $y' = x + y, y(1) = 0,$ by Taylor's series method. Find y(1.1). **(8)**
5. Find the Taylor series solution with three terms for the initial value problem $y' = x^3 + y, y(1) = 1.$ **(8)**
6. Using Taylor series method with the first five terms in the expansion find y(0.1) correct to three decimal places, given that $y' = e^x - y^2, y(0) = 1.$ **(8)**
7. Using Taylor series method, find y(1.1) and y(1.2) correct to four decimal places given. $\frac{dy}{dx} = xy^{1/3}$ and y(1) =1. **(8)**

8. Using Euler's method find y(0.2) and y(0.4) from $y' = x + y, y(0) = 1,$ with h=0.2. **(8)**
9. Using Euler's method find y(0.1) from $y' = x + y + xy, y(0) = 1,$ with h=0.05. **(6)**
10. Using Euler's method find y(0.3)of y(x) satisfies the initial value problem. $y' = \frac{1}{2}(x^2 + 1)y^2, y(0.2) = 1.1114,$ with h=0.2. **(8)**
11. Using modified Euler's method, compute y(0.1) with h=0.1 from $\frac{dy}{dx} = y - \frac{2x}{y}, y(0) = 1.$ **(8)**
12. Solve $y' = 1 - y, y(0) = 0,$ by modified Euler's method. **(8)**

13. Given $\frac{dy}{dx} = x^3 + y, y(0) = 2.$ Compute y(0.2), y(0.4) and y(0.6) by Runge-Kutta method of fourth order. **(8)**

14. Using R.K. method of 4[th] order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}, y(0) = 1.$ at x = 0.2. **(8)**

15. Using R.K. method of fourth order find y(0.1) for the initial value problem $\frac{dy}{dx} = \frac{xy}{1+x^2}, y(0) = 1.$, take h=0.1 **(8)**

16. Find y(0.8) given that $y' = y - x^2, y(0.6) = 1.7379,$ by using R.K. method of fourth order. **(8)**

17. Consider the second order initial value problem $y'' - 2y' + 2y = e^{2t} \sin t$ with y(0) = -0.4 and $y'(0) = -0.6$ using fourth order R.K.method, find y(0.2). **(8)**

18. Apply the fourth order Runge-Kutta method , to find an approximate value of y when x = 0.2 and x= 0.4, given that $y' = x + y, y(0) = 1,$ with h=0.2. **(8)**

19. Given $\frac{dy}{dx} = x^3 + y, y(0) = 2.$ The values of y(0.2) = 2.073,y(0.4)=2.452 and y(0.6) = 3.023 are got by R.K. method of fourth order. Find y(0.8) by Milne's predictor-corrector method taking h=0.2. **(8)**

20. Using Milne's method find y(4.4) given $5xy' + y - 2 = 0$ given y(4)=1, y(4.1) = 1.0049,y(4.2)=1.0097 and y(4.3) = 1.0143. **(8)**

21. Determine the value of y(0.4) using Milne's method given $\frac{dy}{dx} = y^2 + xy, y(0) = 1;.$ use Taylor series to get the values of y(0.1), y(0.2) and y(0.3). **(16)**

22. Using Runge-Kutta method of order 4, find y for x=0.1,0.2,0.3 given that $\frac{dy}{dx} = y^2 + xy, y(0) = 1;.$ and also find the solution at x=0.4 using Milne's method. **(16)**

23. Given $\frac{dy}{dx} = x^2(1+y), y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979,$ evaluate y(1.4) by Adams – Bashforth method. **(8)**

24. Consider the initial value problem $\frac{dy}{dx} = y - x^2 + 1, y(0) = 0.5.$ **(16)**

(i) Using the modified Euler method, find y(0.2)

(ii) Using 4th order Runge-Kutta method, find y(0.4) and y(0.6)

(iii) Using Adam-Bashforth Predictor- Corrector method. Find y(0.8).

25. Using the above predictor-corrector equations, evaluate y(.4) , if y satisfies $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$ and y(1) = 1 , y(1.1) = 0.996, Y91.3) = 0.972 **(8)**

UNIT V - BOUNDARY VALUE PROBLEMS IN ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS

Part – A

1. State the conditions for the equation.
   A$u_{xx} + Bu_{yy} + Cu_{yy} + Du_x + Eu_y + Fu = G$ where A,B,C,D,E,F,G are function of x and y to be
   (i) elliptic (ii) parabolic (iii) hyperbolic.
2. State the condition for the equation A$u_{xx} + Bu_{yy} + Cu_{yy} = f(u_x, u_y, x, y)$ to be
   (i) elliptic (ii) parabolic (iii) hyperbolic when A,B,C are funby Bction of x and y.
3. What is the classification of $f_x - f_{yy} = 0$?
4. Give an exaple of a parabolic equation.
5. State Schmidt's explicit formula for solving heat flow equation.
6. Write an explicit formula to solve numerically the heat equation (parabolic equation) $u_{xx} - au_t = 0$.
7. Write down the crank Nicolson formula to solve $u_{xx} = u_t$.
8. Write the diagonal five point formula to solve Laplace equation $u_{xx} + u_{yy} = 0$
9. Write down the implicit formula to solve one dimensional heat flow equation $u_{xx} = \frac{1}{c^2} u_t$.
10. What is the value of k to solve $\frac{\partial u}{\partial t} = \frac{1}{2} u_{xx}$ by Bender-Schmidt method with h=1 if h and k are the increments of x and t respectively?

11. Write down the finite difference scheme for solving the poison equation $\nabla^2 u = 0.$
12. What is Liebmann's principle?
13. What is Shooting method ?
14. Write different methods for solving B.V.P.
15. Define a difference quotient.

PART – B

1. By Cranck-Nicholson method solve the equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ subject to u(x,0)=0, u(0,t) = 0 and u(1,t)=t for two time steps. **(8)**
2. Solve $\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}$, 0<x<1.t>0 given u(x,0)=0 $\frac{\partial}{\partial t}(x,0)$ =u(0,t)=0 and u(1,t)=100sinπt, complete u(x,t) for 4 times steps with h=0.25 **(8)**
3. Solve the equation $y'' = x + y$ with boundary conditions y(0)=y(1)=0 numerically taking Δx=0.25**.** **(8)**
4. Solve $\nabla^2 u = -10(x^2 + y^2 + 10.)$ over the square mesh with sides x=0,y=0,x=3,y=3 with u=0 on the boundary and mesh length of 1 unit**.** **(8)**
5. Solve $\frac{\partial^2 u}{\partial x^2} = 2\frac{\partial u}{\partial t}$ given u(0,t)=0, u(4,t)=0,u(x,0)=x(4-x) taking Δx=Δt=1. Find the value of u upto t=3 using Bender-Schmidt explicit difference scheme **.** **(8)**

6. Solve the Laplace's equation over the square mesh of side 4 units satisfying the boundary conditions**:** **(16)**

   U(0,y)=0,0$\le y \le 4$;u(4,y)=12+y, 0$\le y \le 4$

   U(x,0)=3x, 0$\le x \le 4$;u(x,4)= x^2,0$\le x \le 4$
7. Derive Bender-Schmidt recurrence formula for solving $u_{xx} - au_t = 0$ with the boundary conditionu(0,t)=T_0, u(l,t)= T_l,u(x,0)=f(x) for 0<x<l Also find corresponding recurrence relation**.** **(16)**

8. Solve the Poisson equation $u_{xx} + u_{yy} = -x^2 y^2$ over the square region bounded by the lines x=0,y=3 given that u=10 throughout the boundaries taking h=1. **(16)**
9. Solve the following Poisson equation over the square region bounded by the lines x=0,y=0,x=3,y=3 given that u=0 throughout the boundaries taking h=1
   i) $\nabla^2 u = -(x+y)^2$ ii) $\nabla^2 u = x^3 + y^3$ **(16)**
10. Solve the Poisson's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -10(x^2 + y^2 + 10)$ over the square with sides x=0=y, x=3=y with u=0 on the boundary and mesh length is 1. **(16)**
11. Obtain a finite difference scheme to solve the Laplace equation. Solve $\nabla^2 u = 0$ at the pivotal posssints in the square shown fitted with square mesh. Use Leibmann's iteration procedure. (5 iteration only) **(16)**

	1000	1000	1000
1000		U_1	U_2
1000		U_3	U_4
1000			
	500	0	0

12. Solve Δ^2u=0, the boundary conditions are given below.(give only three iteration) **(16)**

	10	20	30B
D0		U_1	U_2
A60		U_3	U_4
	60	60	60C

13. Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ in 0<x<5, t $t \geq 0$ given that u(x,0) =20, u(0,t)=0,u(5,t) =100. Compute u for the time-step with h=1 by Cranck – Nicholson method.

(8)

14. Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$,0<x<2,t>0,u(0,t)=u(2,t)=0,t>0 and u(x,0)= $\sin\frac{\pi x}{2}$,0 ≤ x ≤ 2, and Δt=0.25 and Δx=0.5 for two times steps by Crank-Nicholson implicit finite difference method.

(8)

15. Using Schmidt's process solve 25 $u_{xx}=u_t$ where 0<x<1 t>0 with boundary conditions u(0,t)=0=u(10,t); u(x,0)= $\frac{x(10-x)}{25}$ and choosing h=1 and k suitably. Find $u_{i,j}$ for i=1,2,3….9 and j= 1,2,3,4. **(8)**

16. Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, 0 ≤ x ≤ 1, t ≥ 0 with u(x,0)=x(1-x), 0<x<1 and u(0,t)=u(1,t) = 0, for all t >0 using explicit method with Δx=0.2 for 3 time steps. **(8)**

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