KINGS

COLLEGE OF ENGINEERING

DEPARTMENT OF MATHEMATICS

QUESTION BANK

Subject Code :MA1201
Subject Name: Mathematics-III Year/Sem:II/III

UNIT-I
PARTIAL DIFFERENTIAL EQUATIONS
PART-A(2 MARKS)

1. Form the PDE by eliminating a and b from $z = (x^2+a^2)(y^2+b^2)$.
2. Find the PDE of the family of spheres having their centres on the line x=y=z.
3. Form a PDE by eliminating the function from the relation $z = f(\frac{x}{y})$.
4. Form a PDE of eliminating the arbitrary function Φ from Φ(x-y, x+y+z)=0.
5. Find the complete integral of q = 2px.
6. Form the p,d,e with $z = e^y f(x + y)$ as solution.
7. Find the p.d.e of the family of planes with equal intercepts made of x and y axes.
8. Define complete solution.
9. Define general solution.
10. Define particular solution of a p.d.e
11. Find the complete integral of p+q = pq
12. Solve $(D^2 - DD' - 2D'^2)z = 0$
13. Solve $(4D^2+12DD'+9D'^2)z = 0$
14. Find the particular integral of $(D^2 - 3DD' - 4D'^2) z = e^{x+2y}$
15. Find the particular integral of $(D^2 - 3DD' - 2D'^2) z = \cos(x+3y)$
16. Solve $(D_x + D_y)^2 = e^{x+y}$
17. Form the p.d.e by eliminating λ and μ from $(x-\lambda)^2+(y-\mu)^2+z^2=1$
18. Find the solution of $p\sqrt{x}+q\sqrt{y}=\sqrt{z}$
19. Find the general solution of $\frac{\partial^2 z}{\partial x \partial y} = xy$
20. Solve $\frac{\partial^2 u}{\partial x \partial t} = e^{-t}\cos x$ if u = 0 when t = 0 and $\frac{\partial u}{\partial t} = 0$ at x = 0.

PART-B(16 MARKS)

1) (a)Form the PDE by eliminating the arbitrary function from the relation

$f(xy+z^2, x+y+z) = 0$. (8)

(b)Form the PDE by eliminating the arbitrary functions f and g in $z = f(x^3+2y) + g(x^3-2y)$. (8)

2) (a)Find the general solution of the P.D.E.$(mz - ny)p + (nx - lz)q = ly - mn$.(8)

(b)Find the equation of the cone satisfying the equation $xp + yq = z$ and passing through the circle $x^2+y^2+z^2 = 4$. (8)

3) (a)Obtain the complete and general integral of $p^2+q^2 = x + y$ (8)

(b)Find the singular solution of $z = px + qy + \sqrt{p^2 + q^2 + 16}$. (8)

4) (a)Find the complete solution of $9(p^2z + q^2) = 4$. (8)

(b)Solve $p^2 + q^2 = x^2 + y^2$. (8)

5) (a)Solve $z^2 (p^2+q^2) = x^2 + y^2$. (8)

(b) Solve $x(y^2-z^2)p + y(z^2-x^2)q - z(x^2-y^2) = 0$. (8)

6) (a) Solve $(x^2-yz)p + (y^2-zx)q = (z^2-xy)$. (8)

(b) Solve $(y+z)p + (z+x)q = x+y$. (8)

7) (a) Solve $(D^2-DD'-30D'^2)z = xy + e^{6x+y}$. (8)

(b) Solve $(D^3-7DD'^2-6D'^3)z = \cos(x+2y) + x$. (8)

8) (a) Solve $(D^3+D^2D'-DD'^2-D'^3)z = e^x\cos 2y$. (8)

(b)Solve $(D^2-D'^2-3D+3D')z = xy+7$. (8)

9) (a) Solve $(D^2-DD'+D'-1)z = \cos(x+2y)$. (8)

(b) Solve $(D^2+D'^2+2DD'+2D+2D'+1)z = e^{2x+y}$. (8)

10) (a)Solve $(D^2 - 2DD')z = x^3y + e^{2x}$ (8)

(b)Solve $(D^2 - D'^2)z = \sin 2x \sin 3y$ (8)

UNIT-II
FOURIER SERIES
PART-A(2MARKS)

1 State Dirichlets condition

2. If $f(x) = e^{ax}$ is expanded as a Fourier series in $(0,2\pi)$ what is the value of b_n .

3. Does $f(x) = \tan x$ posses a Fourier expansion

4. Obtain the value of a_0 in the Fourier expansion of $f(x) = \sqrt{1-\cos x}$ in $(0,2\pi)$

5. In the Fourier expansion of f(x)= $1+\dfrac{2x}{\pi}$, - π < x < 0

   $= 1-\dfrac{2x}{\pi}$, 0 < x < π in (-π, π).

   Find the value of b_n the coefficient of sin nx.

6. Determine the Value of a_n of the function $f(x) = x^2$ of period 2 π in the Fourier expansion.

7. If f(x) = x + x^2 is expanded as a Fourier series in (-π,π) find the value of a_n

8. If f(x) = $|\sin x|$ is expanded as a Fourier series in (-π, π). Find a_1.

9. If f(x) = x(2π-x) is expanded as a Fourier series in (0,2π). Find a_n

10. Find the Fourier constant b_n when x^2 is expanded as a fourier series in (-π,π).

11. Define root mean square value of a function f(x) in a<x<b.
12. Find the root mean square value of the function f(x) = x in the interval (0,l)
13. What you mean by Harmonic Analysis?
14. State Parseval's Theorem on Fourier series.
15. Write the Complex form of the Fourier series of f(x).
16. State the nature of the Fourier expansion of f(x) = xcosh2x in (-π,π).
17. Examine whether the function f(x) = $\dfrac{1}{1-x}$ can be expanded in a Fourier series in any interval included x = 1.
18. Fourier series of period 2 for $|x|$ in (0,2) contains only cosine terms. Say true or false.
19. Without evaluating any integral , write the half range series with sine terms for $f(x) = \sin^3 x$ in (0,π).
20. To what value, the Fourier series corresponding to f(x) = x^2 in (0,2π) converges at x = 0.

PART-B(16 MARKS)

1. (a) Find the Fourier series $f(x) = 1$ in (0, π)

   = 2 in (π ,2 π)

   and hence find the sum of the $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}$---- ∞ . (8)

   (b)Obtain the Fourier series for $f(x) = 1 + x + x^2$ in the interval -π < x < π.

   Deduce that $\dfrac{1}{1^2}+\dfrac{1}{2^2}+\dfrac{1}{3^2}--- = \dfrac{\pi^2}{6}$ (8)

2. (a)Determine the Fourier series for the function = 1+x , 0 < x < π

   = -1+x , - π < x < 0.

   Hence deduce that $1-\dfrac{1}{3}+\dfrac{1}{5}-\dfrac{1}{7}+--- = \dfrac{\pi}{4}$ (8)

(b)Find the Fourier series expansion of period l for the function

$$f(x) = x \text{ in } (0, l/2)$$
$$= l - x \text{ in } (l/2, l). \qquad (8)$$

3. (a)Find the Fourier series for the function f(x) = x in 0<x<1
   = 1 –x in 1 < x <2

   deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}---- \infty = \frac{\pi^2}{8}$. (8)

   (b)Obtain Fourier series of period $2l$ for f(x) where f(x) $= l - x$ in $0 \le x \le l$
   $= 0$ in $l \le x \le 2l$.

   Hence find the sum of $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+--- \infty$ (8)
4. (a)Find the Fourier series expansion of the periodic function f(x) of period 2l defined by

   f(x) = $l + x$, $-l \le x \le 0$

   l - x, $0 \le x \le l$ deduce that $\sum_{n=1}^{n=\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$. (8)

   (b)Find the Fourier series for f(x) = 0 , -1$\le x \le 0$
   = 1, 0 $\le x \le 1$. (8)
5. (a)Obtain the Fouier series fir the function f(x) = πx , 0 $\le x \le 1$
   = π(2-x), 1$\le x \le 2$ (8)

   (b)Obtain the Fourier expansion of x sin x as a cosine series in (0, π) and

   hence deduce the value of $1+\frac{2}{1.3}-\frac{2}{3.5}+\frac{2}{5.7}.....$ (8)
6. (a)Explain f(x)= (1+ cos x) ^{2} as Foruier cosine series in (0, π) (8)
   (b)Find the half range sine cosine series for the function f(x) = e^x. (8)
7. (a)Find the half range sine and cosine series for the function f(x) = x cos x in (0, π). (8)
   (b)Obtain the half range cosine series for f(x) = (x-2)^2 in the (0, 2). Deduce that
   $\sum_{n=1}^{n=\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$ (8)
8. (a)Find the half range sine series for f(x) = (π –x)^2 in the interval (0, π). Hence

   find the Sum of the series $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}--- \infty$ (8)

   (b)Find the half range cosine series for f(x) = x (π –x) in the interval (0, π) and

   deduce that $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}--- \infty = \frac{\pi^4}{96}$. (8)

   9(a)Find the Fourier series as the second Harmonic to represent the function given in the following data (8)

x	0	1	2	3	4	5
y	9	18	24	28	26	20

(b) Find the 1,2 and 3 fundamental harmonic of the Fourier series of f(x) given by the following table (8)

x	0	1	2	3	4	5
y	4	8	15	7	6	2

10(a).Calculate the first two harmonic of the Fourier series from the following data(8)

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0

(b).Find the Fourier series upto first harmonic (8)

T(sec)	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	2T
A(amp)	1.98	1.3	1.05	1.3	-8.8	-2.5	1.98

UNIT-III
BOUNDARY VALUE PROBLEMS
<u>PART-A</u>(2MARKS)

1.Find the nature of PDE $4u_{xx}+4u_{yy}+u_{yy}+2u_x-u_y=0$

2.Classify the equation $u_{xx}-y^4u_{yy}=2y^3u_y$

3. Classify the p.d.e $(1+x)^2u_{xx}-4xu_{yy}+u_{yy}=x$

4. Classify : $x^2u_{xx}+2xyu_{xy}+(1+y^2)u_{yy}-2u_x = 0$

5. Consider the following partial differential equations

6. Classify the following second order differentiao equations

7. Classify the partial differential equation $u_{xx}+xu_{yy}= 0$

8. Classify the equation : $\frac{\partial^2 u}{\partial x^2}+4\frac{\partial^2 y}{\partial x\partial y}+4\frac{\partial^2 u}{\partial y^2}-12\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+7u = x^2+y^2$

9. State the wave equation and give the various Solutions of it.

10. What are the various Solutions of $\frac{\partial^2 y}{\partial t^2}=a^2\frac{\partial^2 u}{\partial x^2}$

11. A string is stretched and fastened tot wo points l apart. Motion is started by displacing the string into the form $y=y_0\sin\frac{\pi x}{l}$ which it is released at time t=0. Formulate this problem as the boundary value problem.

12. What is the constant a^2 in the wave equation $U_{tt} = a^2u_{xx}$ or In the wave equation $\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2}$ what does c^2 stand for ?

13. State the suitable Solution of the one dimensional heat equation $\frac{\partial u}{\partial t}=a^2\frac{\partial^2 u}{\partial x^2}$

14. State the governing equation for one dimensional heat equation and necessary conditions to solve the problem

15. Write all variable separable Solutions of the one dimensional heat eqauation $u^t=\alpha^2u_{xx}$

16. State any two laws which are assumed to derive one dimensional heat equation.

17. A rod of length 20cm whose one end is kept at 30^0C and the other end is kept at 70^0C is maintained so until steady state prevails. Find the steady state temperature.
18. A bar of length 50cms has its ends kept at 20^0C and 100^0C until steady state conditions prevail. Find the temperature any point of the bar.
19. A rod 30cm long has its ends A and B kept at 20^0C and 80^0C respectively until steady state conditions prevail. Find the steady state temperature in the rod.
20. State two-dimensional Laplace equation

PART-B(16 MARKS)

1. A string is stretched and fastened to 2 points x=0 and x=l . motion is started by displacing the string into the form y=k(lx-x^2) from which it is released at time t=0. Find the displacement of any point on the string at a distance of x from one end at time t. (16)

2. A string of length 2l is fastened at both ends . the mid point of the string is taken to a height b and then released from rest in that position. Find the displacement. (16)

3. A tightly stretched string of length l has its ends fastened at x=0 and x=l . The mid point of the string is then taken to height h and then released from rest in equilibrium position . Find the displacement. Of a point of the string at time 't' from the instant of release. (16)

4. The boundary value problem governing the steady-state temperature distribution in a flat, thin, square plate is given by

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0 \text{ , } 0<x<a, o<y<a,$$

u(x,0)=0,u(x,a)= $4\sin^3\left(\frac{\pi x}{a}\right), 0<x<a,$

u(0,y)=0,u(a,y)=0, 0<y<a.

Find the steady –state temperature distribution in the plate. (16)

5. The ends A and B of a rod l c.m. long have their temperatures kept at $30^{\cdot}$c and $80^{\cdot}$c, until steady state conditions prevail. The temperature of the end B is suddenly reduced to $60^{\cdot}$c and that of A is increased to $40^{\cdot}$c . Find the temperature distribution in the rod after time t. (16)

6. If a string of length l is initially at rest in its equilibrium position and each of its points is given a velocity v such that

V= cx for 0<x<l/2

= c(l - x) for l/2 <x <l

show that the displacement at any time t is given by

$$y(x,t)= \frac{4l^2c}{\pi^3 a}\left[\sin\frac{\pi x}{l}\sin\frac{\pi at}{l}-\frac{1}{3^3}\sin\frac{3\pi x}{l}\sin\frac{3\pi at}{l}+....\right] \qquad (16)$$

7. A rectangular plate with insulated surfaces is 'a' cm wide and so long compared to its width that it may be considered infinite in length, x=a and the short edge at infinity are kept at temperature $0^{\cdot}$c, while the other short edge y=0 is kept at temperature $u_0 \sin^3\left(\frac{\pi x}{a}\right)$, , find the steady state temperature at any point (x,y) of the plate. (16)
8. Find the steady state temperature at any point of a square plate if two adjacent edges are kept at $0^{\cdot}$c and the others at $100^{\cdot}$c (16)
9. A rectangular plate with insulated surface is 10cm wide and so long compared to its width that it may be considered infinite in length without introducing appreciable error. The temperature at short edge y=0 is given by

$$U = \begin{cases} 20x & \text{for } o<x<5 \\ 20(10-x) & \text{for } 5<x<10 \end{cases}$$

   and all the other three edges are kept at 0c . Find the steady state temperature at any point in the plate. (16)

10. Find the steady state temperature distribution in a rectangular plate of sides a and b insulated at the lateral surface and satisfying the boundary conditions u(0,y)=u(a,y)=0 for o<y<b, u(x,b)=0 and u(x,0)=x(a-x) for 0<x<a. (16)
11. A tightly stretched flexible string has its ends fixed at x=0 and x=l . At time t=0 , the string is given a shape defined by $f(x)=kx^2(l-x)$, where k is a constant , and then released from rest. Find the displacement of any point x of the string at any time t>0. (16)
12. The ends A and B of a rod lcm long have the temperatures 40c and 90c until steady state prevails. The temperature at A is suddenly raised to 90c and at the same time that at B is lowered to 40c . Find the temperature distribution in the rod at time t. Also show that the temperature at the mid point of the rod remains unaltered for all time, regarless of the material of the rod. (16)

13. The ends A and B of a rod *30* c.m. long have their temperatures kept at $20^{\cdot}$c and $80^{\cdot}$c, until steady state conditions prevail. The temperature of the end B is suddenly reduced to $60^{\cdot}$c and that of A is increased to $40^{\cdot}$c . Find the temperature distribution in the rod after time t. (16)

14 .A rectangular plate with insulated surface is 10cm wide and so long

compared to its width that it may be considered infinite in length without introducing appreciable error. The temperature at short edge x=0 is given by

$$U = \begin{cases} 20y & \text{for } o<y<5 \\ 20(10-y) & \text{for } 5<y<10 \end{cases}$$

and all the other three edges are kept at 0c . Find the steady state temperature at any point in the plate. (16)

15. The points of trisection of a string are pulled aside through a distance b on opposite sides of the position of equilibrium and the string is released from rest. Find an expression for the displacement. (16)
16. A tightly stretched string of length 2l is fixed at both ends. The midpoint of the string is displaced by a distance "b" transversely and the string is released from rest in this position. Find the displacement of any point of the string at any subsequent time. (16)
17. An infinitely long uniform plate is bounded by two parallel edges and an end at right angle to them. The breadth of this edge x=0 is π, this end is maintained at temperature as $u=K(\pi y-y^2)$ at all points while the other edges are at zero temperature. Find the temperature u(x,y) at any point of the plate in the steady state. (16)

18. A bar of 10cm long, with insulated sides has its ends A and B maintained at temperatures 50^0C and 100^0C respectively, until steady-state conditions prevail. The temperature at A is suddenly raised to 90^0C and at B is lowered to 60^0C . Find the temperature distribution in the bar thereafter. (16)
19. A tightly stretched string with fixed end points x=0 and x=L, is initially in its equilibrium position. If it is set vibrating giving each velocity 3x(L-x), find the displacement. (16)
20. rod of length l has its end A and B kept at 0^0C and 100^0C respectively. Until steady state conditions prevail. If the temperature at B is reduced suddenly to 75^0C and at the same time the temperature at A raised to 25^0C find the temperature u(x,t) at a distance x from A and at time t. (16)

UNIT-IV
FOURIER TRANSFORM
PART-A (2MARKS)

1. State Fourier integral theorem
2. Show that f(x) = 1, 0<x<∞ cannot be represented by a Fourier Integral.
3. Define Fourier Transform pair
4. Find the Fourier transform of f(x) if

$$f(x) = \begin{cases} 1 \; ; |x|<a \\ 0 \; ; |x|>a>0 \end{cases}$$

5. Define the Fourier transform and its inverse transform

6. What is the Fourier cosine transform of a function

   Find the Fourier cosine transform of

$$f(x) = \begin{cases} \text{Cos if } 0<x<a \\ 0 \quad \text{if } x \geq a \end{cases}$$

7. Find the Fourier cosine transform of e^{-ax}, a>0
8. Find the Fourier cosine transform of e^{-3x}
9. Find Fourier cosine transform of e^{-x}
10. Find the Fourier sine transform of e^{-3x}
11. Define Fourier sine transform and its inversion formula

12. If F(s) is the Fourier transform of f(x) , then show that the Fourier transform of $e^{iax}f(x)$ is F(s+a).
13. State the convolution theorem for Fourier transforms
14. State the Fourier transform of the derivatives of a function.
15. Prove that F_c[f(x)cosax] = $\frac{1}{2}\left[F_c(s+a)+F_c(s-a)\right]$ where F_c denotes the Fourier cosine transform f(x).

16. If F(s) if the complex Fourier transform of f(x) then find F[f(x-a)]
18. If F_c(s) is the Fourier cosine transform of f(x). Prove that the Fourier cosine transform of f(ax) is $\frac{1}{a}F_c\left[\frac{s}{a}\right]$
19. If F(s) is the Fourier transform of f(x) , then find the Fourier transform of f(x-a).
20. If F_s(s) is the Fourier sine transform of f(x) , show that F_s(f(s)cosax) = $\frac{1}{2}\left[F_s(s+a)+F_s(s-a)\right]$

PART-B(16 MARKS)

1. (a)Find the Fourier cosine transform of e^{-4x}. Deduce that

   $\int_0^\infty \frac{\cos 2x}{x^2+16}dx = \frac{\pi}{8}e^{-8}$ and $\int_0^\infty \frac{x\sin 2x}{x^2+16}dx = \frac{\pi}{2}e^{-8}$ (8)

   (b)Find the Fourier transform of

$$f(x) = \begin{cases} 1 & \text{for } /x/ < 1 \\ 0 & \text{otherwise} \end{cases}$$

   Hence prove that $\int_0^\infty \frac{\sin x}{x}dx = \int_0^\infty \frac{\sin^2 x}{x^2}dx = \frac{\pi}{2}$ (8)

2. (a)Find the Fourier Sine transform of

$$f(x) = \begin{cases} \sin x, \quad 0<x<\pi \end{cases}$$

0 π,x<∞ (8)

(b)Prove that $e^{\frac{-x^2}{2}}$ is self reciprocal under Fourier Cosine transform. (8)

3. (a)Find the Fourier transform of $e^{-a/x/}$, a>0. Hence deduce that

$$F\left(xe^{-a|x|}\right)=i\sqrt{\frac{2}{\pi}}\frac{2as}{\left(a^2+s^2\right)^2}$$ (8)

(b)Solve for f(x) from the integral equation $\int_0^\infty f(x)\cos\alpha x dx = e^{-\alpha}$ (8)

4. (a)Find the Fourier sine transform of e^{-ax}, a>0 and hence deduce the inversion formula. (8)

(b)Find the Fourier Sine transform of

$$f(x) = \begin{cases} \sin x, & 0<x<a \\ 0 & x>a \end{cases}$$ (8)

5. (a)Find the Fourier transform of f(x) given by f(x) = $1-x^2$ for /x/ ≤1
0 for /x/ ≥1

Hence evaluate $\int_0^\infty \left[\frac{\sin x - x\cos x}{x^3}\right]\cos\left(\frac{x}{2}\right)dx$ (8)

(b)Find the Fourier transform of f(x) if

$$f(x) = \begin{cases} 1-/x/ & \text{for } /x/<1 \\ 0 & \text{for } /x/>1 \end{cases}$$ (8)

6 (a) If F[f(x)] = $\overline{f}(s)$ $prove that F[f(ax)] = \frac{1}{|a|}\overline{f}\left(\frac{s}{a}\right)$ (8)

(b) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$, where a>0. (8)

7.(a) Find Fourier Cosine transform of $e^{-a^2x^2}$ and hence find Fourier sine transform of x $e^{-a^2x^2}$ (8)

(b) Use transform method to evaluate $\int_0^\infty \frac{dx}{\left(x^2+1\right)\left(x^2+4\right)}$ (8)

8. (a) Find the Fourier sine transform of

$$f(x) = \begin{cases} 1-x^2, 0<x<1 \\ 0, \text{otherwise} \end{cases}$$

Hence prove that $\int_0^\infty \left[\frac{\sin x - x\cos x}{x^3}\right]\cos\left(\frac{x}{2}\right)dx = \frac{3\pi}{16}$. (8)

(b). Find the Fourier transform of

$$f(x) = \begin{cases} X & \text{for } /x/ \leq a \\ 0 & \text{for } /x/ > a \end{cases}$$ (8)

9.(a) Find Fourier cosine transform e^{-x^2} (8)

(b) Find the Fourier sine transform of $e^{-|x|}$.Hence show that

$$\int_0^\infty \left[\frac{x\sin x}{(1+x)^3}\right]dx = \frac{\pi}{2}e^{-a}, m > 0$$ (8)

10. (a)Find Fourier sine transform and cosine transform of e^{-x} and hence find the Fourier sine transform of $\frac{x}{(1+x)^2}$ and Fourier cosine transform of $\frac{1}{(1+x)^2}$ (8)

(b). Find the Fourier sine transform of x $xe^{-x^2/2}$ (8)

UNIT-V
Z-TRANSFORM
PART-A (2MARKS)

1. Define Z- Transforms.
2. Define unit step function and unit impulse function
3. Prove that $Z\left[a^n\right] = \frac{z}{z-a}$ and deduce that $z\left[1\right]$
4. Find the $Z\left[\frac{1}{n(n+1)}\right]$
5. Prove that $Z\left[nf(n)\right] = -z\frac{dF(z)}{dz}$
6. Find $Z\left[\frac{a^n}{n!}\right]$
7. Find $Z\left[\cos n\theta\right]$ and $Z\left[\sin n\theta\right]$
8. Find $Z\left[e^t \sin 2t\right]$
9. Find $Z\left[f(n+1)\right] = Z F(z) - z f(0)$
10. Find the Z-transform of $\left(n_{c_k}\right)$
11. Find $Z[a^n n]$
12. Prove that $Z(n) = \frac{z}{(z-1)^2}$

13. Find $[a^{n-1}]$
14. Define Convolution of sequences
15. Find $Z[(-1)^n]$
16. Find Z(t) We know that $Z\{f(t)\} = \sum_{n=0}^{\infty} f(nT)z^{-n}$
17. Find $Z[a^{n-1}]$
18. Find $Z(n^2)$
19. Find the Z transform of $na^n u(n)$
20. Find $Z[e^{at+b}]$

PART-B(16 MARKS)

1. (a)Using Z- Transform solve the equation $u_{n+2} + 3u_{n+1} + 2u_n = 0$ given $u(0) = 1$ and $u(1) = 2$. (8)

   (b)Using Z- Transform solve the equation $u_{n+2} - 5u_{n+1} + 6u_n = 4^n$ given $u(0) = 0$ and $u(1) = 1$. (8)

2. (a)Using Z- Transform solve the equation $y_{n+2} + 4y_{n+1} - 5y_n = 24n - 8$ given $y(0) = 3$ and $y(1) = -5$. (8)

   (b)Find $Z^{-1}\left[\frac{z(z^2 - z + 2)}{(z+1)(z-1)^2}\right]$ by using method of partial fraction. (8)

3. (a)Find $Z\left[\frac{1}{(n+1)(n+2)}\right]$ by using method of partial fraction. (8)

   (b)Using Convolution theorem evaluate $Z^{-1}\left[\frac{z^2}{(z-1)(z-3)}\right]$ (8)

4. (a)Using Convolution theorem evaluate $Z^{-1}\left[\frac{8z^2}{(2z-1)(4z+1)}\right]$ (8)

   (b)State and Prove Convolution theorem on Z-transforms (8)

5. (a)State and Prove initial value and Final value theorem. (8)

   (b)Find $Z^{-1}\left[\frac{9z^3}{(3z-1)^2(z-2)}\right]$ by using residue method. (8)

6. (a)Find $Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right]$ by using Convolution theorem (8)

   (b) Find $Z\left[a^n r^n \cos n\theta\right]$ and $Z\left[a^n r^n \sin n\theta\right]$ (8)

7. (a)Prove that $Z\left[\frac{1}{(n+1)}\right] = z \log\left[\frac{z}{z-1}\right]$ (8)

(b) Find $Z\left[\sinh(t+T)\right]$ (8)

8. (a)Find $Z\left[t^k\right]$ deduce that $Z\left[t^2\right]$. (8)

(b) Find $Z^{-1}\left[\frac{20z}{(z-1)(z-2)}\right]$ (8)

9. (a)Find $Z^{-1}\left[\frac{2z^2-10z+13}{(z-3)^2(z-2)}\right]$ when $2<|z|<3$ (8)

(b)Derive the difference equation from $y_n= (A+Bn)(-3)^n$ (8)

10.(a)Derive the difference equation from $u^n=A2^n+Bn$ (8)

(b)Solve $y_{n+1}-2y_n=0$ given $y_0=3$ (8)

**********************ALL THE BEST********************************